Chapter

1

Overview of Satellite Systems

1.1 Introduction

The use of satellites in communications systems is very much a fact of everyday life, as is evidenced by the many homes which are equipped with antennas, or "dishes," used for reception of satellite television. What may not be so well known is that satellites form an essential part of telecommunications systems worldwide, carrying large amounts of data and telephone traffic in addition to television signals.

Satellites offer a number of features not readily available with other means of communications. Because very large areas of the earth are visible from a satellite, the satellite can form the star point of a communications net linking together many users simultaneously, users who may be widely separated geographically. The same feature enables satellites to provide communications links to remote communities in sparsely populated areas which are difficult to access by other means. Of course, satellite signals ignore political boundaries as well as geographic ones, which may or may not be a desirable feature.

To give some idea of cost, the construction and launch costs of the Canadian Anik-E1 satellite (in 1994 Canadian dollars) were $281.2 million, and the Anik-E2, $290.5 million. The combined launch insurance for both satellites was $95.5 million. A feature of any satellite system is that the cost is *distance insensitive,* meaning that it costs about the same to provide a satellite communications link over a short distance as it does over a large distance. Thus a satellite communications system is economical only where the system is in continuous use and the costs can be reasonably spread over a large number of users.

Satellites are also used for remote sensing, examples being the detection of water pollution and the monitoring and reporting of weather conditions. Some of these remote sensing satellites also form a vital link in search and rescue operations for downed aircraft and the like.

A good overview of the role of satellites is given by Pritchard (1984) and Brown (1981). To provide a general overview of satellite systems here, three different types of applications are briefly described in this chapter: (1) the largest international system, Intelsat, (2) the domestic satellite system in the United States, Domsat, and (3) U.S. National Oceanographic and Atmospheric Administration (NOAA) series of polar orbiting satellites used for environmental monitoring and search and rescue.

1.2 Frequency Allocations for Satellite Services

Allocating frequencies to satellite services is a complicated process which requires international coordination and planning. This is carried out under the auspices of the International Telecommunication Union. To facilitate frequency planning, the world is divided into three regions:

Region 1: Europe, Africa, what was formerly the Soviet Union, and Mongolia

Region 2: North and South America and Greenland

Region 3: Asia (excluding region 1 areas), Australia, and the southwest Pacific

Within these regions, frequency bands are allocated to various satellite services, although a given service may be allocated different frequency bands in different regions. Some of the services provided by satellites are

Fixed satellite service (FSS)

Broadcasting satellite service (BSS)

Mobile satellite services

Navigational satellite services

Meteorological satellite services

There are many subdivisions within these broad classifications; for example, the fixed satellite service provides links for existing telephone networks as well as for transmitting television signals to cable companies for distribution over cable systems. Broadcasting satellite services are intended mainly for direct broadcast to the home, sometimes referred to as *direct broadcast satellite* (DBS) service [in Europe it may be known as *direct-to-home* (DTH) service]. Mobile satellite services would include land mobile, maritime mobile, and aeronautical mobile. Navigational satellite services include global positioning systems, and satellites intended for the meterorological services often provide a search and rescue service.

Table 1.1 lists the frequency band designations in common use for satellite services. The Ku band signifies the band *u*nder the K band, and the Ka band is the band *a*bove the K band. The Ku band is the one used at present for direct broadcast satellites, and it is also used for certain fixed satellite services. The C band is used for fixed satellite services, and no direct broadcast services are allowed in this band. The VHF band is used for certain mobile and navigational services and for data transfer from weather satellites. The L band is used for mobile satellite services and navigation systems. For the fixed satellite service in the C band, the most widely used subrange is approximately 4 to 6 GHz. The higher frequency is nearly always used for the uplink to the satellite, for reasons which will be explained later, and common practice is to denote the C band by 6/4 GHz, giving the uplink frequency first. For the direct broadcast service in the Ku band, the most widely used range is approximately 12 to 14 GHz, which is denoted by 14/12 GHz. Although frequency assignments are made much more precisely, and they may lie somewhat outside the values quoted here (an example of assigned frequencies in the Ku band is 14,030 and 11, 730 MHz), the approximate values stated above are quite satisfactory for use in calculations involving frequency, as will be shown later in the text.

Care must be exercised when using published references to frequency bands because the designations have developed somewhat differently for radar and communications applications; in addition, not all countries use the same designations. The official ITU frequency

TABLE 1.1 Frequency Band Designations

Frequency range, GHz	Band designation
0.1–0.3	VHF
0.3–1.0	UHF
1.0–2.0	L
2.0–4.0	S
4.0–8.0	C
8.0–12.0	X
12.0–18.0	Ku
18.0–27.0	K
27.0–40.0	Ka
40.0–75	V
75–110	W
110–300	mm
300–3000	μm

band designations are shown in Table 1.2 for completeness. However, in this text the designations given in Table 1.1 will be used, along with 6/4 GHz for the C band and 14/12 GHz for the Ku band.

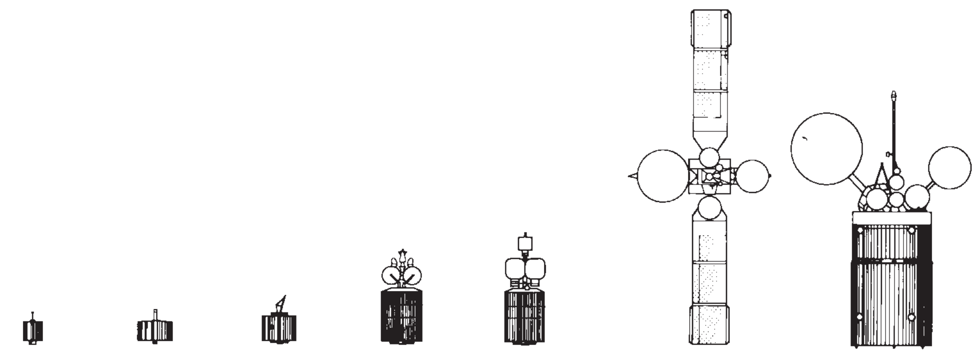
1.3 INTELSAT

INTELSAT stands for *International Telecommunications Satellite.* The organization was created in 1964 and currently has over 140 member countries and more than 40 investing entities (see *http://www.intelsat.com/* for more details). Starting with the Early Bird satellite in 1965, a succession of satellites has been launched at intervals of a few years. Figure 1.1 illustrates the evolution of some of the INTELSAT satellites. As the figure shows, the capacity, in terms of number of voice channels, increased dramatically with each succeeding launch, as well as the design lifetime. These satellites are in *geostationary orbit,* meaning that they appear to be stationary in relation to the earth. The geostationary orbit is the topic of Chap. 3. At this point it may be noted that geostationary satellites orbit in the earth's equatorial plane and that their position is specified by their longitude. For international traffic, INTELSAT covers three main regions, the Atlantic Ocean Region (AOR), the Indian Ocean Region (IOR), and the Pacific Ocean Region (POR). For each region, the satellites are positioned in geostationary orbit above the particular ocean, where they provide a transoceanic telecommunications route. The coverage areas for INTELSAT VI are shown in Fig. 1.2. Traffic in the AOR is about three times that in the IOR and about twice that in the IOR and POR combined. Thus the system design is tailored mainly around AOR requirements (Thompson and Johnston, 1983). As of May 1999, there were three INTELSAT VI satellites in service in the AOR and two in service in the IOR.

TABLE 1.2 ITU Frequency Band Designations

Band number	Symbols	Frequency range (lower limit exclusive, upper limit inclusive)	Corresponding metric subdivision	Metric abbreviations for the bands
4	VLF	3–30 kHz	Myriametric waves	B.Mam
5	LF	30–300 kHz	Kilometric waves	B.km
6	MF	300–3000 kHz	Hectometric waves	B.hm
7	HF	3–30 MHz	Decametric waves	B.dam
8	VHF	30–300 MHz	Metric waves	B.m
9	UHF	300–3000 MHz	Decimetric waves	B.dm
10	SHF	3–30 GHz	Centimetric waves	B.cm
11	EHF	30–300 GHz	Millimetric waves	B.mm
12		300–3000 GHz	Decimillimetric waves	

SOURCE: ITU Geneva.



Designation: Intelsat	I	II	III	IV	IV A	V	V A/V B	VI
Year of first launch	1965	1966	1968	1971	1975	1980	1984/85	1986/87
Prime contractor	Hughes	Hughes	TRW	Hughes	Hughes	Ford Aerospace	Ford Aerospace	Hughes
Width (m)	0.7	1.4	1.4	2.4	2.4	2.0	2.0	3.6
Height (m)	0.6	0.7	1.0	5.3	6.8	6.4	6.4	6.4
Launch vehicles		Thor Delta		Atlas-Centaur		Atlas-Centaur and Ariane	Atlas-Centaur and Ariane	STS and Ariane
Spacecraft mass in transfer orbit (kg)	68	182	293	1385	1489	1946	2140	12,100/3720
Communications payload mass (kg)	13	36	56	185	190	235	280	800
End-of-life (EOL) power of equinox (W)	40	75	134	480	800	1270	1270	2200
Design lifetime (years)	1.5	3	5	7	7	7	7	10
Capacity (number of voice channels)	480	480	2400	8000	12,000	25,000	30,000	80,000
Bandwidth (MHz)	50	130	300	500	800	2137	2480	3520

Figure 1.1 Evolution of INTELSAT satellites. (*From Colino 1985; courtesy of ITU Telecommunications Journal.*)

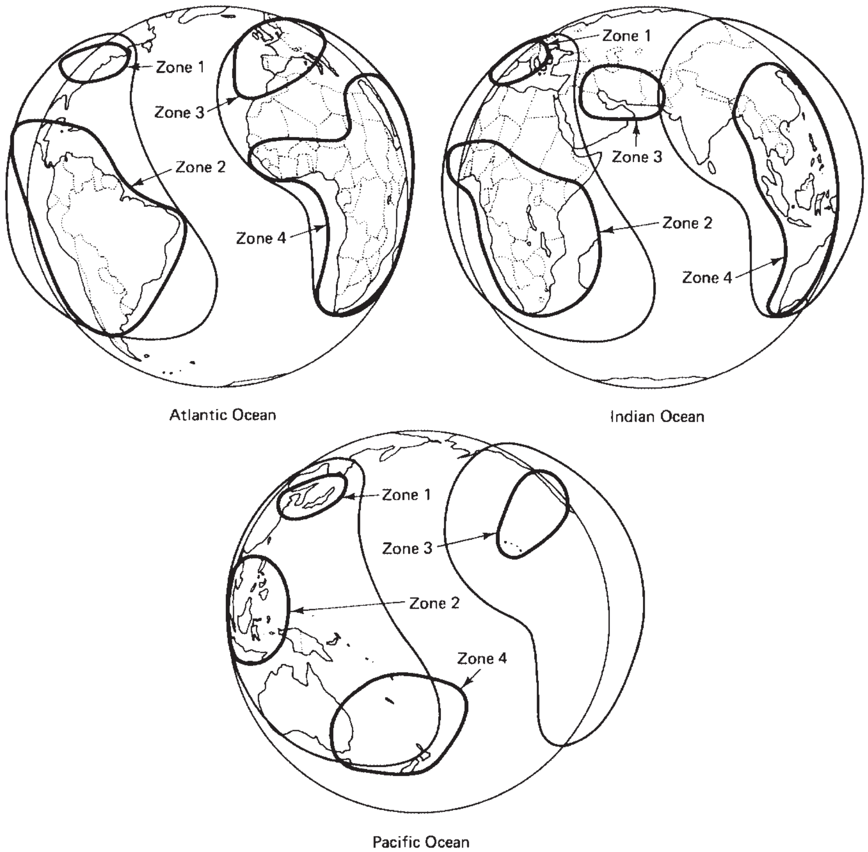


Figure 1.2 INTELSAT VI coverage areas. (*From P. T. Thompson and E. C. Johnston, INTELSAT VI: A New Satellite Generation for 1986–2000, International Journal of Satellite Communications, vol. 1, 3–14. © John Wiley & Sons, Ltd.*)

The INTELSAT VII-VII/A series was launched over a period from October 1993 to June 1996. The construction is similar to that for the V and VA/VB series shown in Fig. 1.1 in that the VII series has solar sails rather than a cylindrical body. This type of construction is described more fully in Chap. 7. The VII series was planned for service in the POR and also for some of the less demanding services in the AOR. The antenna beam coverage is appropriate for that of the POR. Figure 1.3 shows the antenna beam footprints for the C-band hemispheric coverage and zone coverage, as well as the spot beam coverage possible with the Ku-band antennas (Lilly, 1990; Sachdev et al., 1990). When used in the AOR, the VII series satellite is inverted north for south (Lilly, 1990), minor adjustments then being needed only to optimize the antenna patterns for this region. The lifetime of these satel-

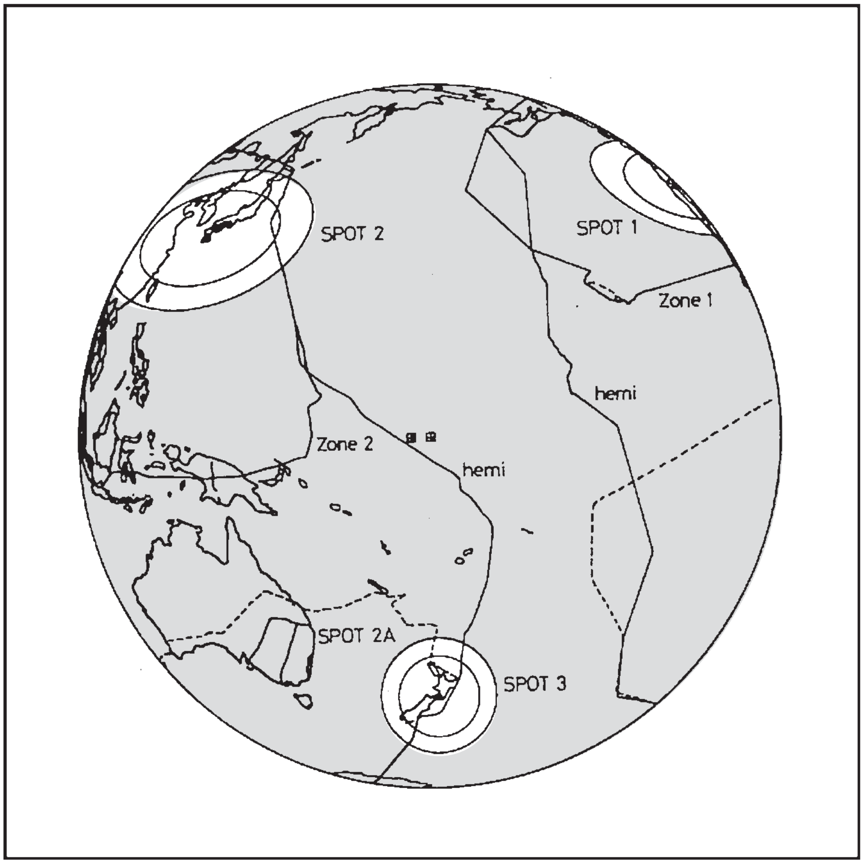


Figure 1.3 INTELSAT VII coverage (Pacific Ocean Region; global, hemispheric, and spot beams). (*From Lilly, 1990, with permission.*)

lites ranges from 10 to 15 years depending on the launch vehicle. Recent figures from the INTELSAT Web site give the capacity for the INTELSAT VII as 18,000 two-way telephone circuits and 3 TV channels; up to 90,000 two-way telephone circuits can be achieved with the use of "digital circuit multiplication." The INTELSAT VII/A has a capacity of 22,500 two-way telephone circuits and 3 TV channels; up to 112,500 two-way telephone circuits can be achieved with the use of digital circuit multiplication. As of May 1999, four satellites were in service over the AOR, one in the IOR, and two in the POR.

The INTELSAT VIII-VII/A series of satellites was launched over a period February 1997 to June 1998. Satellites in this series have similar capacity as the VII/A series, and the lifetime is 14 to 17 years.

It is standard practice to have a spare satellite in orbit on high-reliability routes (which can carry preemptible traffic) and to have a ground spare in case of launch failure. Thus the cost for large international schemes can be high; for example, series IX, described below, represents a total investment of approximately $1 billion.

The INTELSAT IX satellites are the latest in the series (Table 1.3). They will provide a much wider range of services than previously and promise such services as Internet, direct-to-home (DTH) TV, telemedicine, tele-education, and interactive video and multimedia.

In addition to providing transoceanic routes, the INTELSAT satellites are also used for domestic services within any given country and regional services between countries. Two such services are Vista for telephone and Intelnet for data exchange. Figure 1.4 shows typical Vista applications.

1.4 U.S. Domsats

Domsat is an abbreviation for *domestic satellite*. Domestic satellites are used to provide various telecommunications services, such as voice, data, and video transmissions, within a country. In the United States, all domsats are situated in geostationary orbit. As is well known, they make available a wide selection of TV channels for the home entertainment market, in addition to carrying a large amount of commercial telecommunications traffic.

U.S. Domsats which provide a direct-to-home television service can be classified broadly as high power, medium power, and low power (Reinhart, 1990). The defining characteristics of these categories are shown in Table 1.4.

The main distinguishing feature of these categories is the equivalent isotropic radiated power (EIRP). This is explained in more detail in Chap. 12, but for present purposes it should be noted that the upper limit of EIRP is 60 dBW for the high-power category and 37 dBW for the low-power category, a difference of 23 dB. This represents an increase in received power of $10^{2.3}$ or about 200:1 in the high-power category, which allows much smaller antennas to be used with the receiver. As noted in

TABLE 1.3 INTELSAT Series IX Geostationary Satellites

Satellite	Projected location	Capacity	Launch window
901	62°E	Up to 96 units of 36 MHz	First quarter 2001
902	60°E	Up to 96 units of 36 MHz	First quarter 2001
903	335.5°E	Up to 96 units of 36 MHz	Second quarter 2001
904	342°E	Up to 96 units of 36 MHz	Third quarter 2001
905	332.5°E	Up to 96 units of 36 MHz	Fourth quarter 2001 to first quarter 2002
906	332.5°E	Up to 92 units of 36 MHz	To be determined
907	328.5°E	Up to 96 units of 36 MHz	To be determined

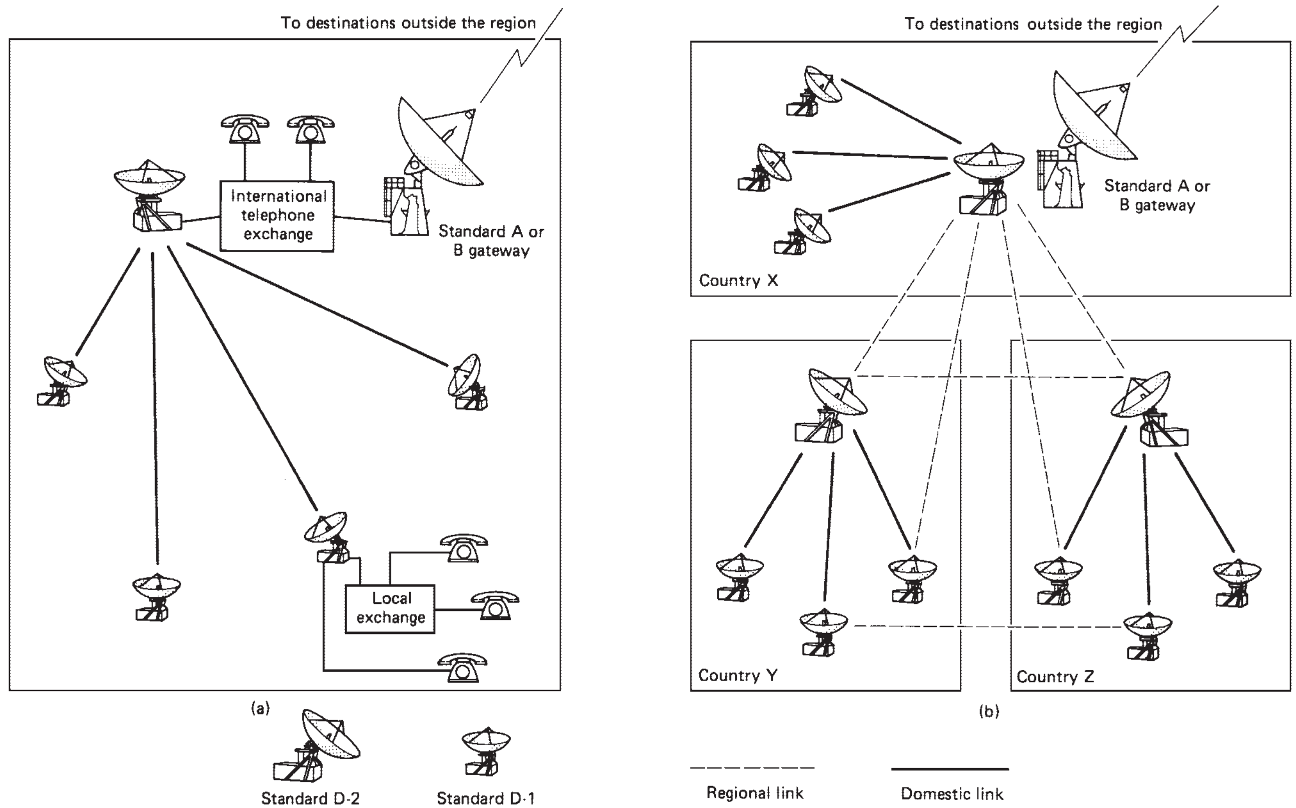


Figure 1.4 (*a*) Typical Vista application; (*b*) domestic/regional Vista network with standard A or B gateway. (*From Colino, 1985; courtesy of ITU Telecommunication Journal.*)

TABLE 1.4 Defining Characteristics of Three Categories of United States DBS Systems

	High power	Medium power	Low power
Band	Ku	Ku	C
Downlink frequency allocation, GHz	12.2–12.7	11.7–12.2	3.7–4.2
Uplink frequency allocation, GHz	17.3–17.8	14–14.5	5.925–6.425
Space service	BSS	FSS	FSS
Primary intended use	DBS	Point to point	Point to point
Allowed additional use	Point to point	DBS	DBS
Terrestrial interference possible	No	No	Yes
Satellite spacing, degrees	9	2	2–3
Satellite spacing determined by	ITU	FCC	FCC
Adjacent satellite interference possible?	No	Yes	Yes
Satellite EIRP range, dBW	51–60	40–48	33–37

ITU: International Telecommunication Union; FCC: Federal Communications Commission.
SOURCE: Reinhart, 1990.

the table, the primary purpose of satellites in the high-power category is to provide a DBS service. In the medium-power category, the primary purpose is point-to-point services, but space may be leased on these satellites for the provision of DBS services. In the low-power category, no official DBS services are provided. However, it was quickly discovered by home experimenters that a wide range of radio and TV programming could be received on this band, and it is now considered to provide a de facto DBS service, witness to which is the large number of TV receive-only (TVRO) dishes which have appeared in the yards and on the rooftops of homes in North America. TVRO reception of C-band signals in the home is prohibited in many other parts of the world, partly for aesthetic reasons because of the comparatively large dishes used, and partly for commercial reasons. Many North American C-band TV broadcasts are now encrypted, or scrambled, to prevent unauthorized access, although this also seems to be spawning a new underground industry in descramblers.

As shown in Table 1.4, true DBS service takes place in the Ku band. Figure 1.5 shows the components of a direct broadcasting satellite system (Government of Canada, 1983). The television signal may be relayed over a terrestrial link to the uplink station. This transmits a very narrowbeam signal to the satellite in the 14-GHz band. The satellite retransmits the television signal in a wide beam in the 12-GHz frequency band. Individual receivers within the beam coverage area will receive the satellite signal.

Table 1.5 shows the orbital assignments for domestic fixed satellites for the United States (FCC, 1996). These satellites are in geostationary orbit, which is discussed further in Chap. 3. Table 1.6 shows the

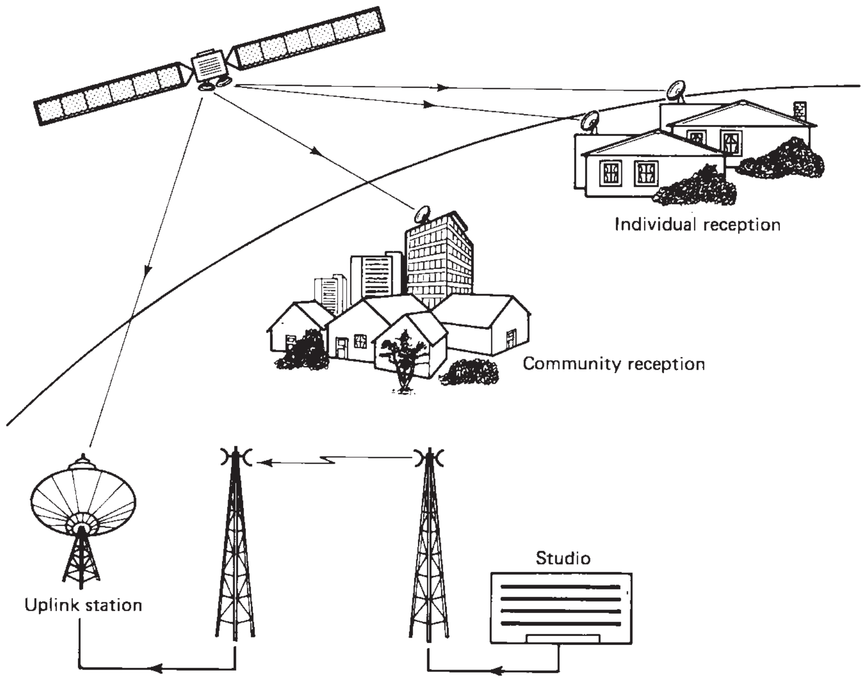


Figure 1.5 Components of a direct broadcasting satellite system. (*From Government of Canada, 1983, with permission.*)

U.S. Ka-band assignments. Broadband services such as Internet (see Chap. 15) can operate at Ka-band frequencies. In 1983, the U.S. Federal Communications Commission (FCC) adopted a policy objective setting 2° as the minimum obital spacing for satellites operating in the 6/4-GHz band and 1.5° for those operating in the 14/12-GHz band (FCC, 1983). It is clear that interference between satellite circuits is likely to increase as satellites are positioned closer together. These spacings represent the minimum presently achievable in each band at acceptable interference levels. In fact, it seems likely that in some cases home satellite receivers in the 6/4-GHz band may be subject to excessive interference where 2° spacing is employed.

1.5 Polar Orbiting Satellites

Polar orbiting satellites orbit the earth in such a way as to cover the north and south polar regions. (Note that the term *polar orbiting* does *not* mean that the satellite orbits around one or the other of the poles). Figure 1.6 shows a polar orbit in relation to the geostationary orbit. Whereas there is only one geostationary orbit, there are, in theory, an

TABLE 1.5 FCC Orbital Assignment Plan (May 7, 1996)

Location	Satellite	Band/polarization
139°W.L.	Aurora II/Satcom C-5	4/6 GHz (vertical)
139°W.L.	ACS-3K (AMSC)	12/14 GHz
137°W.L.	Satcom C-1	4/6 GHz (horizontal)
137°W.L.	Unassigned	12/14 GHz
135°W.L.	Satcom C-4	4/6 GHz (vertical)
135°W.L.	Orion O-F4	12/14 GHz
133°W.L.	Galaxy 1-R(S)	4/6 GHz (horizontal)
133°W.L.	Unassigned	12/14 GHz
131°W.L.	Satcom C-3	4/6 GHz (vertical)
131°W.L.	Unassigned	12/14 GHz
129°W.L.	Loral 1	4/6 GHz (horizontal)/12/14 GHz
127°W.L.	Galaxy IX	4/6 GHz (vertical)
127°W.L.	Unassigned	12/14 GHz
125°W.L.	Galaxy 5-W	4/6 GHz (horizontal)
125°W.L.	GSTAR II/unassigned	12/14 GHz
123°W.L.	Galaxy X	4/6 GHz (vertical)/12/14 GHz
121°W.L.	EchoStar FSS-2	12/14 GHz
105°W.L.	GSTAR IV	12/14 GHz
103°W.L.	GE-1	4/6 GHz (horizontal)
103°W.L.	GSTAR 1/GE-1	12/14 GHz
101°W.L.	Satcom SN-4 (formerly Spacenet IV-n)	4/6 GHz (vertical)/12/14 GHz
99°W.L.	Galaxy IV(H)	4/6 GHz (horizontal)/12/14 GHz
97°W.L.	Telstar 401	4/6 GHz (vertical)/12/14 GHz
95°W.L.	Galaxy III(H)	4/6 GHz (horizontal)/12/14 GHz
93°W.L.	Telstar 5	4/6 GHz (vertical)
93°W.L.	GSTAR III/Telstar 5	12/14 GHz
91°W.L.	Galaxy VII(H)	4/6 GHz (horizontal)/12/14 GHz
89°W.L.	Telestar 402R	4/6 GHz (vertical)/12/14 GHz
87°W.L.	Satcom SN-3 (formerly Spacenet III-R)/GE-4	4/6 GHz (horizontal)/12/14 GHz
85°W.L.	Telstar 302/GE-2	4/6 GHz (vertical)
85°W.L.	Satcom Ku-1/GE-2	12/14 GHz
83°W.L.	Unassigned	4/6 GHz (horizontal)
83°W.L.	EchoStar FSS-1	12/14 GHz
81°W.L.	Unassigned	4/6 GHz (vertical)
81°W.L.	Satcom Ku-2/ unassigned	12/14 GHz
79°W.L.	GE-5	4/6 GHz (horizontal)/12/14 GHz
77°W.L.	Loral 2	4/6 GHz (vertical)/12/14 GHz
76°W.L.	Comstar D-4	4/6 GHz (vertical)
74°W.L.	Galaxy VI	4/6 GHz (horizontal)
74°W.L.	SBS-6	12/14 GHz
72°W.L.	Unassigned	4/6 GHz (vertical)
71°W.L.	SBS-2	12/14 GHz
69°W.L.	Satcom SN-2/Telstar 6	4/6 GHz (horizontal)/12/14 GHz
67°W.L.	GE-3	4/6 GHz (vertical)/12/14 GHz
64°W.L.	Unassigned	4/6 GHz (horizontal)
64°W.L.	Unassigned	12/14 GHz
62°W.L.	Unassigned	4/6 GHz (vertical)
62°W.L.	ACS-2K (AMSC)	12/14 GHz
60°W.L.	Unassigned	4/6 GHz
60°W.L.	Unassigned	12/14 GHz

NOTES: FCC: Federal Communications Commission; W.L.: west longitude; E.L.: east longitude.

TABLE 1.6 Ka-Band Orbital Assignment Plan (FCC, May 9, 1997)

Location	Company	Band
147°W.L.	Morning Star Satellite Company, L.L.C.	20/30 GHz
127°W.L.	Under consideration	20/30 GHz
125°W.L.	PanAmSat Licensee Corporation	20/30 GHz
121°W.L.	Echostar Satellite Corporation	20/30 GHz
115°W.L.	Loral Space & Communications, LTD.	20/30 GHz
113°W.L.	VisionStar, Inc.	20/30 GHz
109.2°W.L.	KaStar Satellite Communications Corp.	20/30 GHz
105°W.L.	GE American Communications, Inc.	20/30 GHz
101°W.L.	Hughes Communications Galaxy, Inc.	20/30 GHz
99°W.L.	Hughes Communications Galaxy, Inc.	20/30 GHz
97°W.L.	Lockheed Martin Corporation	20/30 GHz
95°W.L.	NetSat 28 Company, L.L.C.	20/30 GHz
91°W.L.	Comm, Inc.	20/30 GHz
89°W.L.	Orion Network Systems	20/30 GHz
87°W.L.	Comm, Inc.	20/30 GHz
85°W.L.	GE American Communications, Inc.	20/30 GHz
83°W.L.	Echostar Satellite Corporation	20/30 GHz
81°W.L.	Orion Network Systems	20/30 GHz
77°W.L.	Comm, Inc.	20/30 GHz
75°W.L.	Comm, Inc.	20/30 GHz
73°W.L.	KaStar Satellite Corporation	20/30 GHz
67°W.L.	Hughes Communications Galaxy, Inc.	20/30 GHz
62°W.L.	Morning Star Satellite Company, L.L.C.	20/30 GHz
58°W.L.	PanAmSat Corporation	20/30 GHz
49°W.L.	Hughes Communications Galaxy, Inc.	20/30 GHz
47°W.L.	Orion Atlantic, L.P.	20/30 GHz
21.5°W.L.	Lockheed Martin Corporation	20/30 GHz
17°W.L.	GE American Communications, Inc.	20/30 GHz
25°E.L.	Hughes Communications Galaxy, Inc.	20/30 GHz
28°E.L.	Loral Space & Communications, LTD.	20/30 GHz
30°E.L.	Morning Star Satellite Company, L.L.C.	20/30 GHz
36°E.L.	Hughes Communications Galaxy, Inc.	20/30 GHz
38°E.L.	Lockheed Martin Corporation	20/30 GHz
40°E.L.	Hughes Communications Galaxy, Inc.	20/30 GHz
48°E.L.	Hughes Communications Galaxy, Inc.	20/30 GHz
54°E.L.	Hughes Communications Galaxy, Inc.	20/30 GHz
56°E.L.	GE American Communications, Inc.	20/30 GHz
78°E.L.	Orion Network Systems, Inc.	20/30 GHz
101°E.L.	Hughes Communications Galaxy, Inc.	20/30 GHz
105.5°E.L.	Loral Space & Communications, LTD.	20/30 GHz
107.5°E.L.	Morning Star Satellite Company, L.L.C.	20/30 GHz
111°E.L.	Hughes Communications Galaxy, Inc.	20/30 GHz
114.5°E.L.	GE American Communications, Inc.	20/30 GHz
124.5°E.L.	Hughes Communications Galaxy, Inc.	20/30 GHz
126.5°E.L.	Orion Asia Pacific Corporation	20/30 GHz
130°E.L.	Lockheed Martin Corporation	20/30 GHz
149°E.L.	Hughes Communications Galaxy, Inc.	20/30 GHz
164°E.L.	Hughes Communications Galaxy, Inc.	20/30 GHz
173°E.L.	Hughes Communications Galaxy, Inc.	20/30 GHz
175.25°E.L.	Lockheed Martin Corporation	20/30 GHz

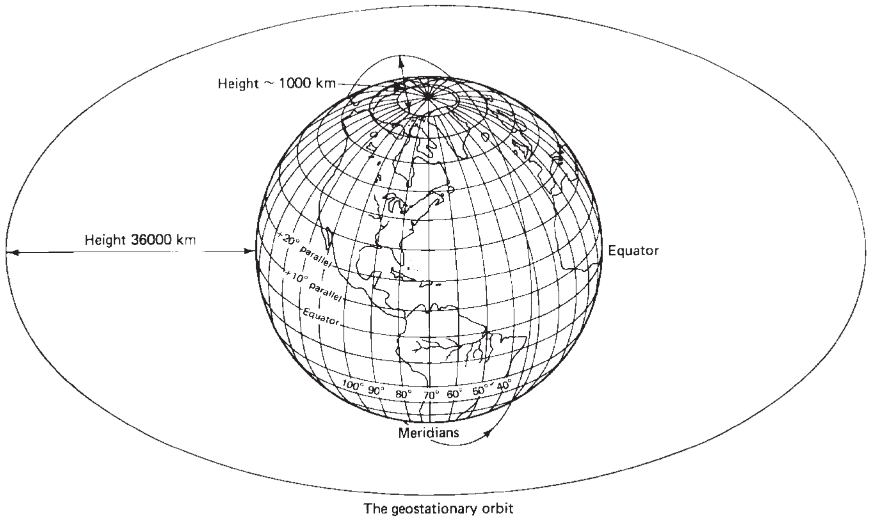


Figure 1.6 Geostationary orbit and one possible polar orbit.

infinite number of polar orbits. The U.S. experience with weather satellites has led to the use of relatively low orbits, ranging in altitude between 800 and 900 km, compared with 36,000 km for the geostationary orbit.

In the United States, the National Oceanic and Atmospheric Administration (NOAA) operates a weather satellite system. Their Web page can be found at *http://www.noaa.gov/*. The system uses both geostationary satellites, referred to as *geostationary operational environmental satellites* (GOES), and *polar operational environmental satellites* (POES). There are two of these polar satellites in orbit at any one time. The orbits are circular, passing close to the poles, and they are *sun synchronous,* meaning that they cross the equator at the same local time each day. The morning orbit, at an altitude of 830 km, crosses the equator going from south to north at 7:30 A.M. each day, and the afternoon orbit, at an altitude of 870 km, at 1:40 P.M. The polar orbiters are able to track weather conditions over the entire earth and provide a wide range of data, including visible and infrared radiometer data for imaging purposes, radiation measurements, and temperature profiles. They carry ultraviolet sensors that measure ozone levels, and they can monitor the ozone hole over Antarctica.

The polar orbiters carry a NOAA letter designation before launch, which is changed to a numeric designation once the satellite achieves orbit. NOAA-J, launched in December 1994, became NOAA-14 in operation. The new series, referred to as the *KLM satellites,* carries much improved instrumentation. Some details are shown in Table 1.7. The

TABLE 1.7 NOAA KLM Satellites

Launch date (callup basis)	NOAA-K (NOAA-15): May 13, 1998 NOAA-L: September 14, 2000 NOAA-M: May 2001 NOAA-N: December 2003 NOAA-N: July 2007
Mission life	2 years minimum
Orbit	Sun-synchronous, 833 ± 19 km or 870 ± 19 km
Sensors	Advanced Very High Resolution Radiometer (AVHRR/3) Advanced Microwave Sounding Unit-A (AMSU-A) Advanced Microwave Sounding Unit-B (AMSU-B) High Resolution Infrared Radiation Sounder (HIRS/3) Space Environment Monitor (SEM/2) Search and Rescue (SAR) Repeater and Processor Data Collection System (DCS/2)

Argos data collection system (DCS) collects environmental data radioed up from automatic data collection platforms on land, on ocean buoys, and aboard free-floating balloons. The satellites process these data and retransmit them to ground stations.

The NOAA satellites also participate in satellite *search and rescue* (SAR) *operations,* known generally as *Cospas-Sarsat,* where Cospas refers to the payload carried by participating Russian satellites and Sarsat to the payloads carried by the NOAA satellites. Sarsat-6 is carried by NOAA-14, and Sarsat-7 is carried by NOAA-15. The projected payloads Sarsat-8 to Sarsat-10 will be carried by NOAA-L to NOAA-N. The Cospas-Sarsat Web page is at *http://www.cospas-sarsat.org/*. As of January 2000, there were 32 countries formally associated with Cospas-Sarsat. Originally, the system was designed to operate only with satellites in low earth orbits (LEOs), this part of the search and rescue system being known as *LEOSAR.* Later, the system was complemented with geostationary satellites, this component being known as *GEOSAR.* Figure 1.7 shows the combined LEOSAR-GEOSAR system.

The nominal space segment of LEOSAR consists of four satellites, although as of January 2000 there were seven in total, three Cospas and four Sarsat. In operation, the satellite receives a signal from an emergency beacon set off automatically at the distress site. The beacon transmits in the VHF/UHF range, at a precisely controlled frequency. The satellite moves at some velocity relative to the beacon, and this results in a Doppler shift in frequency received at the satellite. As the satellite approaches the beacon, the received frequency appears to be higher than the transmitted value. As the satellite recedes from the beacon, the received frequency appears to be lower than the transmitted value. Figure 1.8 shows how the beacon frequency, as received at the satellite, varies for different passes. In all cases, the received frequency goes from

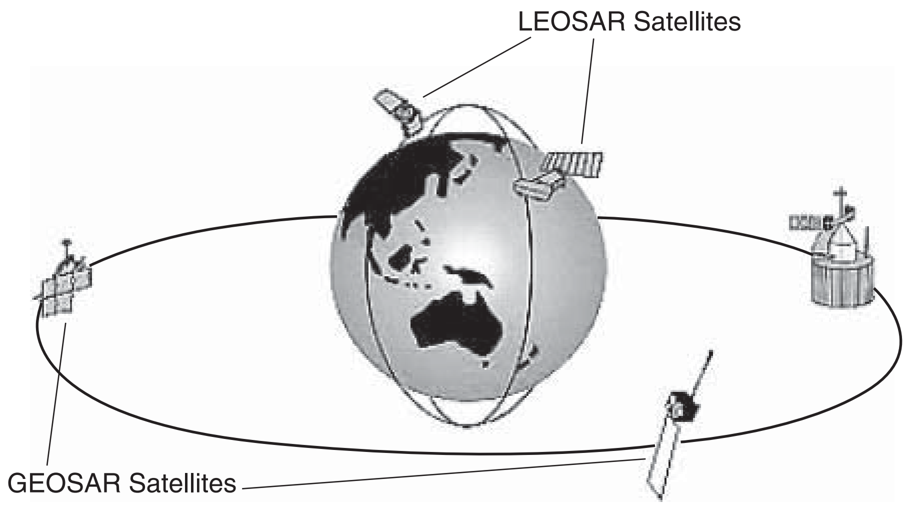


Figure 1.7 Geostationary Orbit Search and Rescue (GEOSAR) and Low Earth Orbit Search and Rescue (LEOSAR) satellites. (*Courtesy Cospas-Sarsat Secretariat.*)

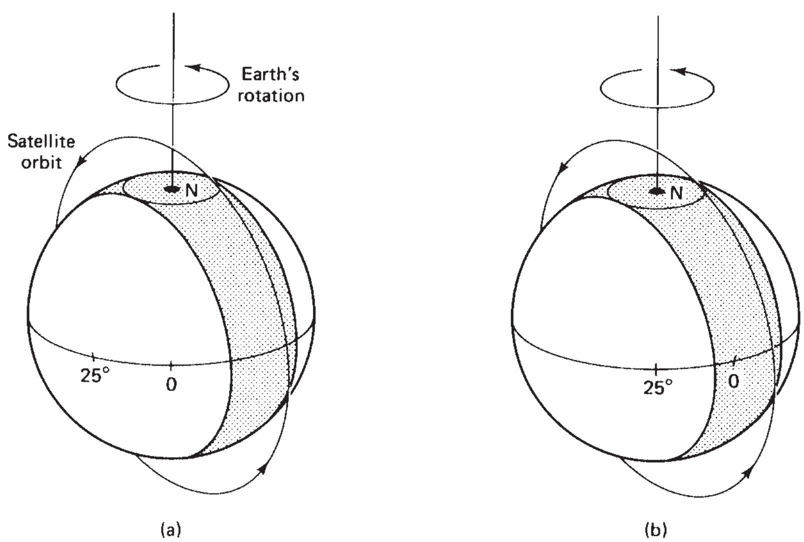


Figure 1.8 Polar orbiting satellite: (*a*) first pass; (*b*) second pass, earth having rotated 25°. Satellite period is 102 min.

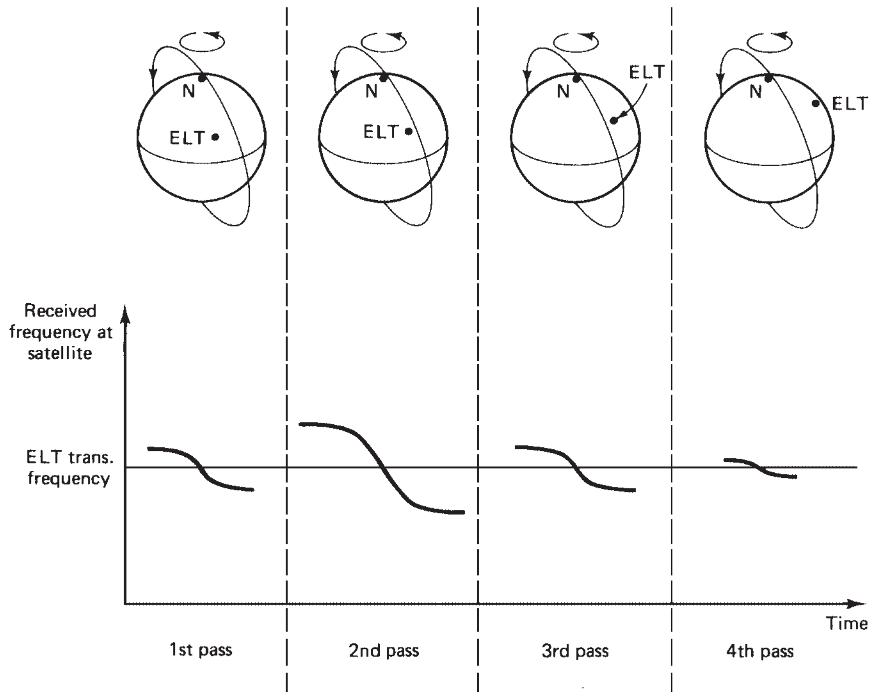


Figure 1.9 Showing the Doppler shift in received frequency on successive passes of the satellite. ELT = emergency locator transmitter.

being higher to being lower than the transmitted value as the satellite approaches and then recedes from the beacon. The longest record and the greatest change in frequency are obtained if the satellite passes over the site, as shown for pass no. 2. This is so because the satellite is visible for the longest period during this pass. Knowing the orbital parameters for the satellite, the beacon frequency, and the Doppler shift for any one pass, the distance of the beacon relative to the projection of the orbit on the earth can be determined. However, whether the beacon is east or west of the orbit cannot be determined easily from a single pass. For two successive passes, the effect of the earth's rotation on the Doppler shift can be estimated more accurately, and from this it can be determined whether the beacon is approaching or receding from the orbital path. In this way, the ambiguity in east-west positioning is resolved. Figure 1.9 illustrates the Doppler shifts for successive passes.

The satellite must of course get the information back to an earth station so that the search and rescue operation can be completed, successfully one hopes. The Sarsat communicates on a downlink frequency of 1544.5 MHz to one of several local user terminals (LUTs) established at various locations throughout the world.

In the original Cospas-Sarsat system, the signal from the emergency locator transmitters (ELTs) was at a frequency of 121.5 MHz. It was found that over 98 percent of the alerts at this frequency were false, often being caused by interfering signals from other services and by inappropriate handling of the equipment. The 121.5-MHz system relies entirely on the Doppler shift, and the carrier does not carry any identification information. The power is low, typically a few tenths of a watt, which limits locational accuracy to about 10 to 20 km. There are no signal storage facilities aboard the satellites for the 121.5-MHz signals, which therefore requires that the distress site (the ELT) and the local user terminal (LUT) must be visible simultaneously from the satellite. Because of these limitations, the 121.5-MHz beacons are being phased out. Cospas-13, planned for launch in 2006, and Sarsat-14, planned for launch from 2009, will not carry 121.5-MHz beacons. However, all Cospas-Sarsat satellites launched prior to these will carry the 121.5-MHz processors. (Recall that Sarsat-7 is NOAA-15, Sarsat-8 is NOAA-L, Sarsat-9 is NOAA-M, and Sarsat-10 is NOAA-N).

The status of the 121.5-MHz LEOSAR system as of January 2000 consisted of repeaters on seven polar orbiters, 35 ground receiving stations (referred to as *LEOSAR local user terminals,* or *LEOLUTs*), and 20 mission control centers (MCCs). The MCC alerts the rescue coordination center (RCC) nearest the location where the distress signal originated, and the RCC takes the appropriate action to effect a rescue. There are about 600,000 distress beacons, carried mostly on aircraft and small vessels.

Newer beacons operating at a frequency of 406 MHz are being introduced. The power has been increased to 5 Watts, which should permit locational accuracy to 3 to 5 km (Scales and Swanson, 1984). These are known as *emergency position indicating radio beacons* (EPIRBs). Units for personnel use are also available, known as *personal locator beacons* (PLBs). The 406-MHz carrier is modulated with information such as an identifying code, the last known position, and the nature of the emergency. The satellite has the equipment for storing and forwarding the information from a continuous memory dump, providing complete worldwide coverage with 100 percent availability. The polar orbiters, however, do not provide continuous coverage. The mean time between a distress alert being sent and the appropriate search and rescue coordination center being notified is estimated at 27 min satellite storage time plus 44 min waiting time for a total delay of 71 min (Cospas-Sarsat, 1994b).

The nominal frequency is 406 MHz, and originally, a frequency of 406.025 MHz was used. Because of potential conflict with the

GEOSTAR system, the frequency is being moved to 406.028 MHz. Beacons submitted for type approval after January 1, 2000 may operate at the new frequency, and after January 1, 2001, all beacons submitted for type approval must operate at a frequency of 406.028 MHz. However, beacon types approved before the January 2001 date and still in production may continue to operate at 406.025 MHz. The power of the 406 MHz beacons is 5 watts.

As shown in Figure 1.7, the overall system incorporates GEOSAR satellites. Because these are stationary, there is no Doppler shift. However, the 406-MHz beacons for the GEOSTAR component carry positional information obtained from the Global Positioning Satellite (GPS) system. The GPS system is described in Chap. 17. It should be noted that the GEOSAR system does not provide coverage of the polar regions.

As mentioned previously, the NOAA satellites are placed in a low earth orbit typified by the NOAA-J satellite. The NOAA-J satellite will orbit the earth in approximately 102.12 min. The orbit is arranged to rotate eastward at a rate of 0.9856°/day, to make it *sun-synchronous.* Sun-synchronous orbits are discussed more fully in Chap. 2, but very briefly, in a sun-synchronous orbit the satellite crosses the same spot on the earth at the same local time each day. One advantage of a sun-synchronous orbit is that the same area of the earth can be viewed under approximately the same lighting conditions each day. By definition, an orbital pass from south to north is referred to as an *ascending pass,* and from north to south, as a *descending pass.* The NOAA-J orbit crosses the equator at about 1:40 P.M. local solar time on its ascending pass and at about 1:40 A.M. local solar time on its descending pass.

Because of the eastward rotation of the satellite orbit, the earth rotates approximately 359° relative to it in 24 h of mean solar time (ordinary clock time), and therefore, in 102.12 min the earth will have rotated about 25.59° relative to the orbit. The satellite "footprint" is displaced each time by this amount, as shown in Fig. 1.7. At the equator, 25.59° corresponds to a distance of about 2848 km. The width of ground seen by the satellite sensors is about 5000 km, which means that some overlap occurs between passes. The overlap is greatest at the poles.

1.6 Problems

1.1. Describe briefly the main advantages offered by satellite communications. Explain what is meant by a *distance-insensitive communications system.*

1.2. Comparisons are sometimes made between satellite and optical fiber communications systems. State briefly the areas of application for which you feel each system is best suited.

1.3. Describe briefly the development of INTELSAT starting from the 1960s through to the present. Information can be found at Web site *http://www.intelsat.com/*.

1.4. From the Web page given above, find the positions of the INTELSAT 7 and the INTELSAT 8 series of satellites, as well as the number of C-band and Ku-band transponders on each.

1.5. From Table 1.5, determine which satellites provide service to each of the regions AOR, IOR, and POR.

1.6. Referring to Table 1.4, determine the power levels, in watts, for each of the three categories listed.

1.7. From Table 1.5, determine typical orbital spacings in degrees for (*a*) the 6/4-GHz band and (*b*) the 14/12-GHz band.

1.8. Give reasons why the Ku band is used for the DBS service.

1.9. An earth station is situated at longitude 91°W and latitude 45°N. Determine the range to the following satellites: (*a*) Galaxy VII, (*b*) Satcom SN-3, and (*c*) Galaxy IV. A spherical earth of uniform mass and mean radius 6371 km may be assumed.

1.10. Given that the earth's equatorial radius is 6378 km and the height of the geostationary orbit is 36,000 km, determine the intersatellite distance between the GE American Communications, Inc., satellite and the Hughes Communications Galaxy, Inc., satellite, operating in the Ka band.

1.11. Explain what is meant by a *polar orbiting satellite*. A NOAA polar orbiting satellite completes one revolution around the earth in 102 min. The satellite makes a north to south equatorial crossing at longitude 90°W. Assuming that the orbit is circular and crosses exactly over the poles, estimate the position of the subsatellite point at the following times after the equatorial crossing: (*a*) 0 h, 10 min; (*b*) 1 h, 42 min; (*c*) 2 h, 0 min. A spherical earth of uniform mass may be assumed.

1.12. By accessing the NOAA Web page at *http://www.noaa.gov/*, find out how the Geostationary Operational Environmental Satellites take part in weather forecasting. Give details of the GOES-10 characteristics.

1.13. The Cospas-Sarsat Web site is at *http://www.cospas-sarsat.org*. Access this site and find out the number and location of the LEOLUTs in current use.

1.14. Using information obtained from the Cospas-Sarsat Web site, find out which satellites carry (*a*) 406-MHz SAR processors (SARPs), (*b*) 406-MHz SAR repeaters (SARRs), and (*c*) 121.5-MHz SAR repeaters. What is the basic difference between a SARP and a SARR?

Chapter

2

Orbits and Launching Methods

2.1 Introduction

Satellites (spacecraft) which orbit the earth follow the same laws that govern the motion of the planets around the sun. From early times much has been learned about planetary motion through careful observations. From these observations Johannes Kepler (1571–1630) was able to derive empirically three laws describing planetary motion. Later, in 1665, Sir Isaac Newton (1642–1727) was able to derive Kepler's laws from his own laws of mechanics and develop the theory of gravitation [for very readable accounts of much of the work of these two great men, see Arons (1965) and Bate et al. (1971)].

Kepler's laws apply quite generally to any two bodies in space which interact through gravitation. The more massive of the two bodies is referred to as the *primary,* the other, the *secondary,* or *satellite.*

2.2 Kepler's First Law

Kepler's first law states that the path followed by a satellite around the primary will be an ellipse. An ellipse has two focal points shown as F_1 and F_2 in Fig. 2.1. The center of mass of the two-body system, termed the *barycenter,* is always centered on one of the foci. In our specific case, because of the enormous difference between the masses of the earth and the satellite, the center of mass coincides with the center of the earth, which is therefore always at one of the foci.

The semimajor axis of the ellipse is denoted by *a,* and the semiminor axis, by *b.* The eccentricity *e* is given by

$$e = \frac{\sqrt{a^2 - b^2}}{} \qquad (2.1)$$

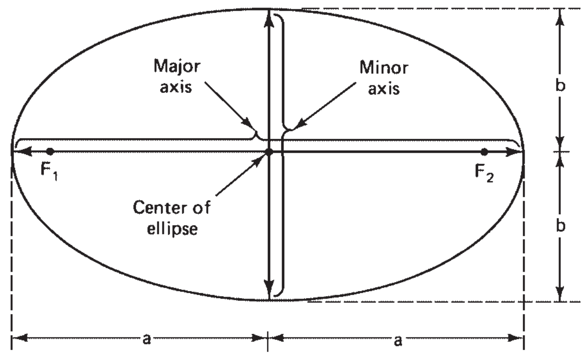


Figure 2.1 The foci F_1 and F_2, the semimajor axis a, and the semiminor axis b of an ellipse.

The eccentricity and the semimajor axis are two of the orbital parameters specified for satellites (spacecraft) orbiting the earth. For an elliptical orbit, $0 < e < 1$. When $e = 0$, the orbit becomes circular. The geometrical significance of eccentricity, along with some of the other geometrical properties of the ellipse, is developed in App. B.

2.3 Kepler's Second Law

Kepler's second law states that, for equal time intervals, a satellite will sweep out equal areas in its orbital plane, focused at the barycenter. Referring to Fig. 2.2, assuming the satellite travels distances S_1 and S_2 meters in 1 s, then the areas A_1 and A_2 will be equal. The average velocity in each case is S_1 and S_2 meters per second, and because of the equal area law, it follows that the velocity at S_2 is less than that at S_1. An important consequence of this is that the satellite takes longer to travel a given distance when it is farther

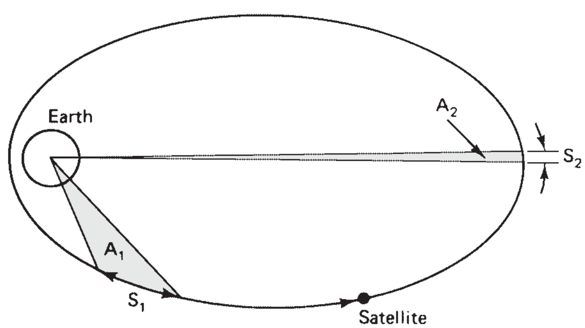


Figure 2.2 Kepler's second law. The areas A_1 and A_2 swept out in unit time are equal.

away from earth. Use is made of this property to increase the length of time a satellite can be seen from particular geographic regions of the earth.

2.4 Kepler's Third Law

Kepler's third law states that the square of the periodic time of orbit is proportional to the cube of the mean distance between the two bodies. The mean distance is equal to the semimajor axis a. For the artificial satellites orbiting the earth, Kepler's third law can be written in the form

$$a^3 = \frac{\mu}{n^2} \tag{2.2}$$

where n is the mean motion of the satellite in radians per second and μ is the earth's geocentric gravitational constant. With a in meters, its value is (see Wertz, 1984, Table L3).

$$\mu = 3.986005 \times 10^{14}\ \mathrm{m^3/sec^2} \tag{2.3}$$

Equation (2.2) applies only to the ideal situation of a satellite orbiting a perfectly spherical earth of uniform mass, with no perturbing forces acting, such as atmospheric drag. Later, in Sec. 2.8, the effects of the earth's oblateness and atmospheric drag will be taken into account.

With n in radians per second, the orbital period in seconds is given by

$$P = \frac{2\pi}{n} \tag{2.4}$$

The importance of Kepler's third law is that it shows there is a fixed relationship between period and size. One very important orbit in particular, known as the *geostationary orbit,* is determined by the rotational period of the earth and is described in Chap. 3. In anticipation of this, the approximate radius of the geostationary orbit is determined in the following example.

Example 2.1 (see App. H for Mathcad notation) Calculate the radius of a circular orbit for which the period is 1-day.

solution The mean motion, in rad/day, is:

$$\mathrm{n} := \frac{2 \cdot \pi}{1\ \mathrm{day}}$$

Note that in Mathcad this will be automatically recorded in rad/s. Thus, for the record:

$$n = 7.272 \cdot 10^{-5} \cdot \frac{\text{rad}}{\text{sec}}$$

The earth's gravitational constant is

$$\mu := 3.986005 \cdot 10^{14} \cdot m^3 \cdot \text{sec}^{-2}$$

Kepler's third law gives

$$a := \left(\frac{\mu}{n^2}\right)^{1/3}$$

$$\underline{\underline{a = 42241 \cdot \text{km}}}$$

Since the orbit is circular the semimajor axis is also the radius.

2.5 Definitions of Terms for Earth-Orbiting Satellites

As mentioned previously, Kepler's laws apply in general to satellite motion around a primary body. For the particular case of earth-orbiting satellites, certain terms are used to describe the position of the orbit with respect to the earth.

Apogee The point farthest from earth. Apogee height is shown as h_a in Fig. 2.3.

Perigee The point of closest approach to earth. The perigee height is shown as h_p in Fig. 2.3.

Line of apsides The line joining the perigee and apogee through the center of the earth.

Ascending node The point where the orbit crosses the equatorial plane going from south to north.

Descending node The point where the orbit crosses the equatorial plane going from north to south.

Line of nodes The line joining the ascending and descending nodes through the center of the earth.

Inclination The angle between the orbital plane and the earth's equatorial plane. It is measured at the ascending node from the equator to the orbit, going from east to north. The inclination is shown as i in Fig. 2.3. It will be seen that the greatest latitude, north or south, is equal to the inclination.

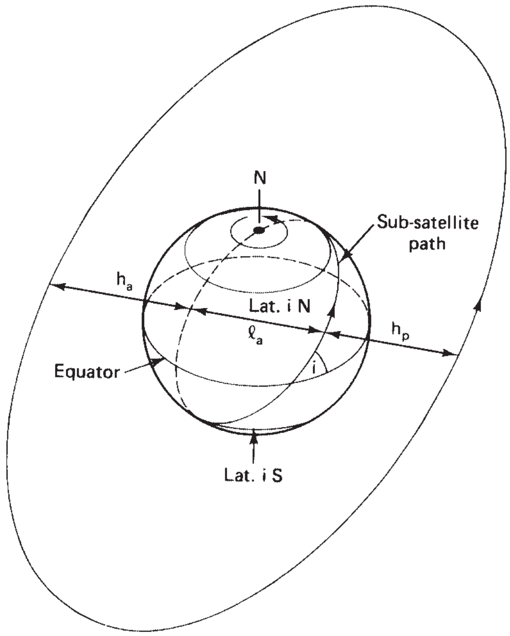


Figure 2.3 Apogee height h_a, perigee height h_p, and inclination *i*. l_a is the line of apsides.

Prograde orbit An orbit in which the satellite moves in the same direction as the earth's rotation, as shown in Fig. 2.4. The prograde orbit is also known as a *direct orbit.* The inclination of a prograde orbit always lies between 0 and 90°. Most satellites are launched in a prograde orbit because the earth's rotational velocity provides part of the orbital velocity with a consequent saving in launch energy.

Retrograde orbit An orbit in which the satellite moves in a direction counter to the earth's rotation, as shown in Fig. 2.4. The inclination of a retrograde orbit always lies between 90 and 180°.

Argument of perigee The angle from ascending node to perigee, measured in the orbital plane at the earth's center, in the direction of satellite motion. The argument of perigee is shown as ω in Fig. 2.5.

Right ascension of the ascending node To define completely the position of the orbit in space, the position of the ascending node is specified. However, because the earth spins, while the orbital plane remains stationary (slow drifts which do occur are discussed later), the longitude of the ascending node is not fixed, and it cannot be used as an absolute reference. For the practical determination of an orbit, the longitude and time of crossing of the ascending node are frequently used. However, for an absolute measurement, a fixed reference in space is required. The reference chosen is the *first point of Aries,* otherwise known as the vernal, or spring, equinox.

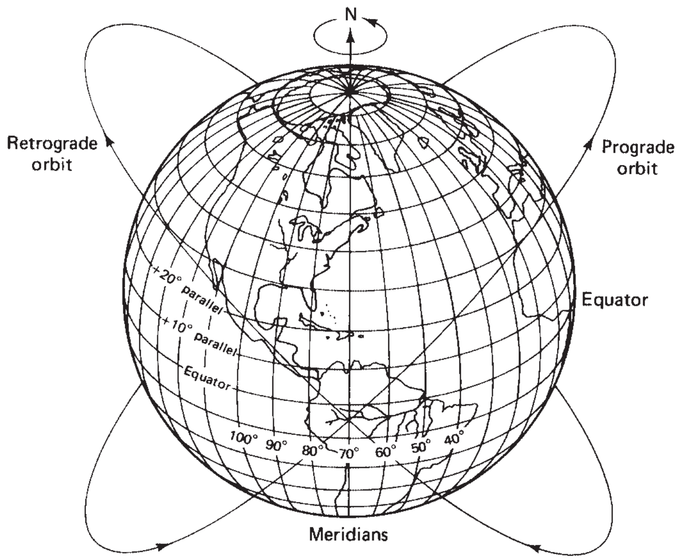


Figure 2.4 Prograde and retrograde orbits.

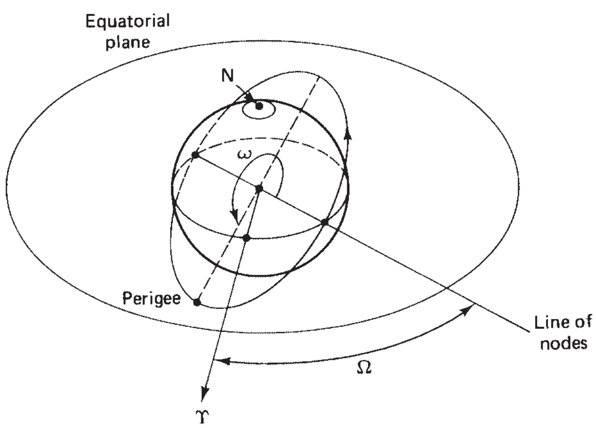


Figure 2.5 The argument of perigee ω and the right ascension of the ascending node Ω.

The vernal equinox occurs when the sun crosses the equator going from south to north, and an imaginary line drawn from this equatorial crossing through the center of the sun points to the first point of Aries (symbol ♈). This is the *line of Aries.* The right ascension of the ascending node is then the angle measured eastward, in the equatorial plane, from the ♈ line to the ascending node, shown as Ω in Fig. 2.5.

Mean anomaly Mean anomaly M gives an average value of the angular position of the satellite with reference to the perigee. For a circular orbit, M gives the angular position of the satellite in the orbit. For elliptical orbit, the position is much more difficult to calculate, and M is used as an intermediate step in the calculation as described in Sec. 2.9.5.

True anomaly The true anomaly is the angle from perigee to the satellite position, measured at the earth's center. This gives the true angular position of the satellite in the orbit as a function of time. A method of determining the true anomaly is described in Sec. 2.9.5.

2.6 Orbital Elements

Earth-orbiting artificial satellites are defined by six orbital elements referred to as the *keplerian element set*. Two of these, the semimajor axis a and the eccentricity e described in Sec. 2.2, give the shape of the ellipse. A third, the mean anomaly M_0, gives the position of the satellite in its orbit at a reference time known as the *epoch*. A fourth, the argument of perigee ω, gives the rotation of the orbit's perigee point relative to the orbit's line of nodes in the earth's equatorial plane. The remaining two elements, the inclination i and the right ascension of the ascending node Ω, relate the orbital plane's position to the earth. These four elements are described in Sec. 2.5.

Because the equatorial bulge causes slow variations in ω and Ω, and because other perturbing forces may alter the orbital elements slightly, the values are specified for the reference time or epoch, and thus the epoch also must be specified.

Appendix C lists the two-line elements provided to users by the U.S. National Aeronautics and Space Administration (NASA). The two-line elements may be downloaded from the Web site shown in Fig. 2.6.

TABLE 2.1 Details from the NASA Bulletins (see Fig. 2.6 and App. C)

Line no.	Columns	Description
1	3–7	*Satellite number:* 25338
1	19–20	*Epoch year* (last two digits of the year): 00
1	21–32	*Epoch day* (day and fractional day of the year): 223.79688452 (this is discussed further in Sec. 2.9.2).
1	34–43	*First time derivative of the mean motion* (rev/day^2): 0.00000307
2	9–16	*Inclination* (degrees): 98.6328
2	18–25	*Right ascension of the ascending node* (degrees): 251.5324
2	27–33	*Eccentricity* (leading decimal point assumed): 0011501
2	35–42	*Argument of perigee* (degrees): 113.5534
2	44–51	*Mean anomaly* (degrees): 246.6853
2	53–63	*Mean motion* (rev/day): 14.23304826
2	64–68	*Revolution number at epoch* (rev/day): 11,663

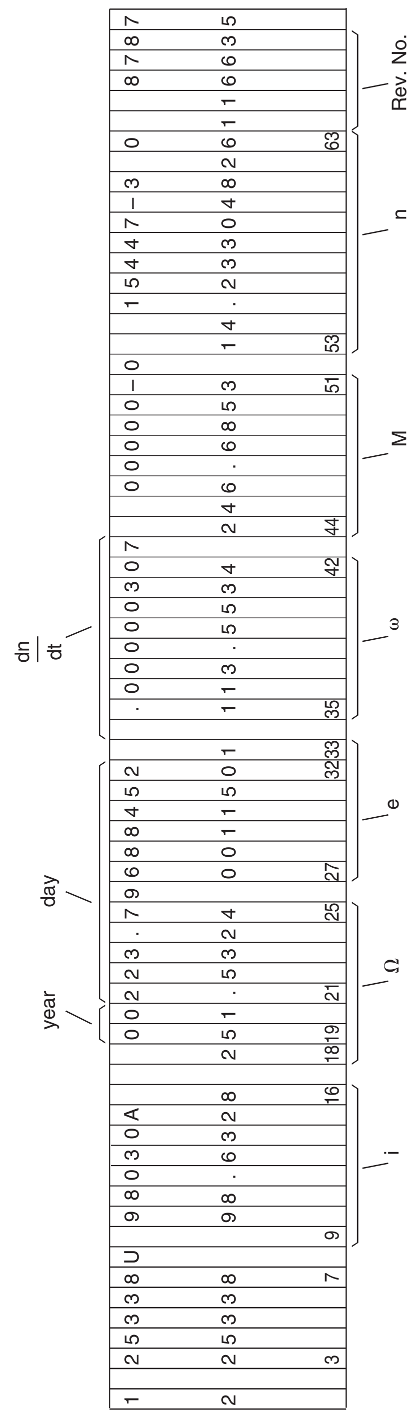


Figure 2.6 Two-line elements for NOAA-15.

It will be seen that the semimajor axis is not specified, but this can be calculated from the data given. An example calculation is presented in Example 2.2.

Example 2.2 Calculate the semimajor axis for the satellite parameters given in Table 2.1.

solution The mean motion is given in Table 2.1 as

$$NN := 14.22296917 \cdot day^{-1}$$

This can be converted to rad/sec as

$$n_0 := NN \cdot 2 \cdot \pi$$

(Note that Mathcad automatically converts time to the fundamental unit of second.) Equation (2.3) gives

$$\mu := 3.986005 \cdot 10^{14} \cdot m^3 \cdot sec^{-2}$$

Kepler's 3rd law gives

$$a := \left(\frac{\mu}{n_o^2}\right)^{1/3}$$

$$a = 7192.3 \cdot km$$
= = = = = = =

2.7 Apogee and Perigee Heights

Although not specified as orbital elements, the apogee height and perigee height are often required. As shown in App. B, the length of the radius vectors at apogee and perigee can be obtained from the geometry of the ellipse:

$$r_a = a\,(1 + e) \tag{2.5}$$

$$r_p = a\,(1 - e) \tag{2.6}$$

In order to find the apogee and perigee heights, the radius of the earth must be subtracted from the radii lengths, as shown in the following example.

Example 2.3 Calculate the apogee and perigee heights for the orbital parameters given in Table 2.1. Assume a mean earth radius of 6371 km.

solution The required data from Table 2.1 are: $e := .0011501$ $a := 7192.3 \cdot km$. (Note that the value for a was determined in Example 2.2.)

Given data:

$$R := 6371 \cdot km$$

$r_a := a \cdot (1 + e)$ ...Eq. (2.5) $r_a = 7200.6 \cdot km$

$r_p := a \cdot (1 - e)$ ...Eq. (2.6) $r_p = 7184.1 \cdot km$

$h_a := r_a - R$ $h_a = 829.6 \cdot km$
= = = = = = = =

$h_p := r_p - R$ $h_p = 813.1 \cdot km$
= = = = = = = =

2.8 Orbit Perturbations

The type of orbit described so far, referred to as a *keplerian orbit,* is elliptical for the special case of an artificial satellite orbiting the earth. However, the keplerian orbit is ideal in the sense that it assumes that the earth is a uniform spherical mass and that the only force acting is the centrifugal force resulting from satellite motion balancing the gravitational pull of the earth. In practice, other forces which can be significant are the gravitational forces of the sun and the moon and atmospheric drag. The gravitational pulls of sun and moon have negligible effect on low-orbiting satellites, but they do affect satellites in the geostationary orbit as described in Sec. 3.5. Atmospheric drag, on the other hand, has negligible effect on geostationary satellites but does affect low-orbiting earth satellites below about 1000 km.

2.8.1 Effects of a nonspherical earth

For a spherical earth of uniform mass, Kepler's third law (Eq. 2.2) gives the nominal mean motion n_0 as

$$n_0 = \sqrt{\frac{\mu}{a^3}} \tag{2.7}$$

The 0 subscript is included as a reminder that this result applies for a perfectly spherical earth of uniform mass. However, it is known that the earth is not perfectly spherical, there being an equatorial bulge and a flattening at the poles, a shape described as an *oblate spheroid.* When the earth's oblateness is taken into account, the mean motion, denoted in this case by symbol n, is modified to (Wertz, 1984).

$$n = n_0 \left[\frac{1 + K_1 (1 - 1.5 \sin^2 i)}{a^2 (1 - e^2)^{1.5}} \right] \tag{2.8}$$

K_1 is a constant which evaluates to 66,063.1704 km^2. The earth's oblateness has negligible effect on the semimajor axis a, and if a is known, the mean motion is readily calculated. The orbital period taking into account the earth's oblateness is termed the *anomalistic period* (e.g., from perigee to perigee). The mean motion specified in the NASA bulletins is the reciprocal of the anomalistic period. The anomalistic period is

$$P_A = \frac{2\pi}{n} \text{ sec} \tag{2.9}$$

where n is in radians per second.

If the known quantity is n (as is given in the NASA bulletins, for example), one can solve Eq. (2.8) for a, keeping in mind that n_0 is also a function of a. Equation (2.8) may be solved for a by finding the root of the following equation:

$$n - \sqrt{\frac{\mu}{a^3}} \left[1 + \frac{K_1 (1 - 1.5 \sin^2 i)}{a^2 (1 - e^2)^{1.5}} \right] = 0 \tag{2.10}$$

This is illustrated in the following example.

Example 2.4 A satellite is orbiting in the equatorial plane with a period from perigee to perigee of 12 h. Given that the eccentricity is 0.002, calculate the semimajor axis. The earth's equatorial radius is 6378.1414 km.

solution Given data:

$$e := .002 \qquad i := 0 \cdot \text{deg} \qquad P := 12 \cdot \text{hr}$$

$$K_1 := 66063.1704 \cdot \text{km}^2 \qquad a_E := 6378.1414 \cdot \text{km}$$

$$\mu := 3.986005 \cdot 10^{14} \cdot \text{m}^3 \cdot \text{sec}^{-2}$$

The mean motion is

$$n := \frac{2 \cdot \pi}{P}$$

Kepler's third law gives

$$a := \left(\frac{\mu}{n^2} \right)^{1/3}$$

$a = 26597 \cdot km$ …This is the nonperturbed value which can be used as a guess value for the root function.
= = = = = =

Perturbed value:

$$a := \text{root}\left[n - \left(\sqrt{\frac{\mu}{a^3}}\right) \cdot \left[1 + \frac{K_1(1 - 1.5 \cdot \sin(i)^2)}{a^2 \cdot (1 - e^2)^{1.5}}\right], a\right]$$

$a = 26598.6 \cdot km$
= = = = = = = =

The oblateness of the earth also produces two rotations of the orbital plane. The first of these, known as *regression of the nodes,* is where the nodes appear to slide along the equator. In effect, the line of nodes, which is in the equatorial plane, rotates about the center of the earth. Thus Ω, the right ascension of the ascending node, shifts its position.

If the orbit is prograde (see Fig. 2.4), the nodes slide westward, and if retrograde, they slide eastward. As seen from the ascending node, a satellite in prograde orbit moves eastward, and in a retrograde orbit, westward. The nodes therefore move in a direction opposite to the direction of satellite motion, hence the term *regression of the nodes.* For a polar orbit ($i = 90°$), the regression is zero.

The second effect is rotation of apsides in the orbital plane, described below. Both effects depend on the mean motion n, the semimajor axis a, and the eccentricity e. These factors can be grouped into one factor K given by

$$K = \frac{nK_1}{\quad} \tag{2.11}$$

K will have the same units as n. Thus, with n in *rad / day,* K will be in *rad / day,* and with n in °/*day,* K will be in °/*day.* An approximate expression for the rate of change of Ω with respect to time is (Wertz, 1984)

$$\frac{d\Omega}{dt} = -K \cos i \tag{2.12}$$

where i is the inclination.

The rate of regression of the nodes will have the same units as n.

When the rate of change given by Eq. (2.12) is negative, the regression is westward, and when the rate is positive, the regression is eastward. It will be seen, therefore, that for eastward regression, i must be greater than 90°, or the orbit must be retrograde. It is possible to choose values of *a, e,* and *i* such that the rate of rotation is 0.9856°/day eastward. Such an orbit is said to be *sun-synchronous* and is described further in Sec. 2.10.

In the other major effect produced by the equatorial bulge, rotation of the line of apsides in the orbital plane, the argument of perigee changes with time, in effect, the rate of change being given by (Wertz, 1984)

$$\frac{d\omega}{dt} = K\,(2 - 2.5 \sin^2 i) \tag{2.13}$$

Again, the units for the rate of rotation of the line of apsides will be the same as those for *n*.

When the inclination *i* is equal to 63.435°, the term within the parentheses is equal to zero, and hence no rotation takes place. Use is made of this fact in the orbit chosen for the Russian Molniya satellites (see Probs. 2.23 and 2.24).

Denoting the epoch time by t_0, the right ascension of the ascending node by Ω_0, and the argument of perigee by ω_0 at epoch gives the new values for Ω and ω at time *t* as

$$\Omega = \Omega_0 + \frac{d\Omega}{dt}\,(t - t_0) \tag{2.14}$$

$$\omega = \omega_0 + \frac{d\omega}{dt}\,(t - t_0 \tag{2.15}$$

Keep in mind that the orbit is not a physical entity, and it is the forces resulting from an oblate earth which act on the satellite to produce the changes in the orbital parameters. Thus, rather than follow a closed elliptical path in a fixed plane, the satellite drifts as a result of the regression of the nodes, and the latitude of the point of closest approach (the perigee) changes as a result of the rotation of the line of apsides. With this in mind, it is permissible to visualize the satellite as following a closed elliptical orbit but with the orbit itself moving relative to the earth as a result of the changes in Ω and ω. Thus, as stated above, the period P_A is the time required to go around the orbital path from perigee to perigee, even though the perigee has moved relative to the earth.

Suppose, for example, that the inclination is 90° so that the regression of the nodes is zero (from Eq. 2.12), and the rate of rotation of the line of apsides is $-K/2$ (from Eq. 2.13), and further, imagine the situation where the perigee at the start of observations is exactly over the ascending node. One period later the perigee would be at an angle $-KP_A/2$ relative to the ascending node or, in other words, would be south of the equator. The time between crossings *at the ascending node* would be $P_A(1 + K/2n)$, which would be the period observed from the earth. Recall that K will have the same units as n, e.g., radians per second.

Example 2.5 Determine the rate of regression of the nodes and the rate of rotation of the line of apsides for the satellite parameters specified in Table 2.1. The value for a obtained in Example 2.2 may be used.

solution from Table 2.1 and Example 2.2:

$$i := 98.6328 \cdot \text{deg} \qquad e := .0011501$$

$$n := 14.23304826 \cdot \text{day}^{-1} \qquad a := 7192.3 \cdot \text{km}$$

Known constant: $K_1 := 66063.1704 \cdot \text{km}^2$

$n := 2 \cdot \pi \cdot n$ …Converts n to SI units of rad/sec.

$$K := \frac{n \cdot K_1}{a^2 \cdot (1 - e^2)^2} \qquad K = 6.544 \cdot \frac{\text{deg}}{\text{day}}$$

$$\Omega' := -K \cdot \cos(i) \qquad \Omega' = 0.982 \cdot \frac{\text{deg}}{\text{day}}$$

= = = = = = =

$$\omega' := K \cdot (2 - 2.5 \cdot \sin(i)^2) \qquad \omega' = -2.903 \cdot \frac{\text{deg}}{\text{day}}$$

= = = = = = = =

Example 2.6 Calculate, for the satellite in Example 2.5, the new values for ω and Ω one period after epoch.

solution From Example 2.5:

$$\Omega' := .982 \cdot \frac{\text{deg}}{\text{day}} \qquad \omega' := -2.903 \cdot \frac{\text{deg}}{\text{day}}$$

From Table 2.1:

$$n := 14.23304826 \cdot \text{day}^{-1} \qquad \omega_O := 113.5534 \cdot \text{deg} \qquad \Omega_0 := 251.5324 \cdot \text{deg}$$

The period is

$$P_A = \frac{1}{n}$$

$$\Omega := \Omega_O + \Omega' \cdot P_A \qquad \Omega = 251.601 \cdot deg$$

$$\omega := \omega_O + \omega' \cdot P_A \qquad \omega = 113.349 \cdot deg$$

In addition to the equatorial bulge, the earth is not perfectly circular in the equatorial plane; it has a small eccentricity of the order of 10^{-5}. This is referred to as the *equatorial ellipticity.* The effect of the equatorial ellipticity is to set up a gravity gradient which has a pronounced effect on satellites in geostationary orbit (Sec. 7.4). Very briefly, a satellite in geostationary orbit ideally should remain fixed relative to the earth. The gravity gradient resulting from the equatorial ellipticity causes the satellites in geostationary orbit to drift to one of two stable points, which coincide with the minor axis of the equatorial ellipse. These two points are separated by 180° on the equator and are at approximately 75° E longitude and 105° W longitude. Satellites in service are prevented from drifting to these points through stationkeeping maneuvers, described in Sec. 7.4. Because old, out-of-service satellites eventually do drift to these points, they are referred to as "satellite graveyards."

It may be noted that the effect of equatorial ellipticity is negligible on most other satellite orbits.

2.8.2 Atmospheric Drag

For near-earth satellites, below about 1000 km, the effects of atmospheric drag are significant. Because the drag is greatest at the perigee, the drag acts to reduce the velocity at this point, with the result that the satellite does not reach the same apogee height on successive revolutions. The result is that the semimajor axis and the eccentricity are both reduced. Drag does not noticeably change the other orbital parameters, including perigee height. In the program used for generating the orbital elements given in the NASA bulletins, a "pseudo-drag" term is generated which is equal to one-half the rate of change of mean motion (ADC USAF, 1980). An approximate expression for the change of major axis is

$$a \cong a_0 \left[\frac{n_0}{n_0 + n_0' (t - t_0)} \right]^{2/3} \qquad (2.16)$$

The mean anomaly is also changed. An approximate expression for the amount by which it changes is

$$\delta M = \frac{n_0'}{2} (t - t_0)^2 \tag{2.17}$$

From Table 2.1 it is seen that the first time derivative of the mean motion is listed in columns 34–43 of line 1 of the NASA bulletin. For the example shown in Fig. 2.6, this is 0.00000307 rev/day^2. Thus the changes resulting from the drag term will be significant only for long time intervals and for present purposes will be ignored. For a more accurate analysis, suitable for long-term predictions, the reader is referred to ADC USAF (1980).

2.9 Inclined Orbits

A study of the general situation of a satellite in an inclined elliptical orbit is complicated by the fact that different parameters are referred to different reference frames. The orbital elements are known with reference to the plane of the orbit, the position of which is fixed (or slowly varying) in space, while the location of the earth station is usually given in terms of the local geographic coordinates which rotate with the earth. Rectangular coordinate systems are generally used in calculations of satellite position and velocity in space, while the earth station quantities of interest may be the azimuth and elevation angles and range. Transformations between coordinate systems are therefore required.

Here, in order to illustrate the method of calculation for elliptical inclined orbits, the problem of finding the earth station look angles and range will be considered. It should be kept in mind that with inclined orbits the satellites are not geostationary, and therefore, the required look angles and range will change with time. Detailed and very readable treatments of orbital properties in general will be found, for example, in Bate et al. (1971) and Wertz (1984). Much of the explanation and the notation in this section is based on these two references.

Determination of the look angles and range involves the following quantities and concepts:

1. The *orbital elements,* as published in the NASA bulletins and described in Sec. 2.6
2. Various measures of *time*
3. The *perifocal coordinate system,* which is based on the orbital plane
4. The *geocentric-equatorial coordinate system,* which is based on the earth's equatorial plane

5. The *topocentric-horizon coordinate system,* which is based on the observer's horizon plane

The two major coordinate transformations which are needed are as follows:

- The satellite position measured in the perifocal system is transformed to the geocentric-horizon system in which the earth's rotation is measured, thus enabling the satellite position and the earth station location to be coordinated.
- The satellite-to-earth station position vector is transformed to the topocentric-horizon system, which enables the look angles and range to be calculated.

2.9.1 Calendars

A calendar is a timekeeping device in which the year is divided into months, weeks, and days. Calendar days are units of time based on the earth's motion relative to the sun. Of course, it is more convenient to think of the sun moving relative to the earth. This motion is not uniform, and so a fictitious sun, termed the *mean sun,* is introduced.

The mean sun does move at a uniform speed but otherwise requires the same time as the real sun to complete one orbit of the earth, this time being the *tropical year.* A day measured relative to this mean sun is termed a *mean solar day.* Calendar days are mean solar days, and generally they are just referred to as days.

A tropical year contains 365.2422 days. In order to make the calendar year, also referred to as the *civil year,* more easily usable, it is normally divided into 365 days. The extra 0.2422 of a day is significant, and for example, after 100 years, there would be a discrepancy of 24 days between the calendar year and the tropical year. Julius Caesar made the first attempt to correct for the discrepancy by introducing the *leap year,* in which an extra day is added to February whenever the year number is divisible by four. This gave the *Julian calendar,* in which the civil year was 365.25 days on average, a reasonable approximation to the tropical year.

By the year 1582, an appreciable discrepancy once again existed between the civil and tropical years. Pope Gregory XIII took matters in hand by abolishing the days October 5 through October 14, 1582, to bring the civil and tropical years into line and by placing an additional constraint on the leap year in that years ending in two zeros must be divisible by 400 to be reckoned as leap years. This dodge was used to miss out 3 days every 400 years. The resulting calendar is the *Gregorian calendar,* which is the one in use today.

Example 2.7 Calculate the average length of the civil year in the Gregorian calendar.

solution The nominal number of days in 400 years is $400 \times 365 = 146{,}000$. The nominal number of leap years is $400/4 = 100$, but this must be reduced by 3, and therefore, the number of days in 400 years of the Gregorian calendar is $146{,}000 + 100 - 3 = 146{,}097$. This gives a yearly average of $146{,}097/400 = 365.2425$.

In calculations requiring satellite predictions, it is necessary to determine whether a year is a leap year or not, and the simple rule is: If the year number ends in two zeros and is divisible by 400, it is a leap year. Otherwise, if the year number is divisible by 4, it is a leap year.

Example 2.8 Determine which of the following years are leap years: (*a*) 1987, (*b*) 1988, (*c*) 2000, (*d*) 2100.

solution

a) 1987/4 = 496.75 (therefore, 1987 is not a leap year)

b) 1988/4 = 497 (therefore, 1988 is a leap year)

(*c*) 2000/400 = 5 (therefore, 2000 is a leap year)

(*d*) 2100/400 = 5.25 (therefore, 2100 is not a leap year)

2.9.2 Universal time

Universal time coordinated (UTC) is the time used for all civil timekeeping purposes, and it is the time reference which is broadcast by the National Bureau of Standards as a standard for setting clocks. It is based on an atomic time-frequency standard. The fundamental unit for UTC is the *mean solar day* [see App. J in Wertz (1984)]. In terms of "clock time," the mean solar day is divided into 24 hours, an hour into 60 minutes, and a minute into 60 seconds. Thus there are 86,400 "clock seconds" in a mean solar day. Satellite-orbit epoch time is given in terms of UTC.

Example 2.9 Calculate the time in days, hours, minutes, and seconds for the epoch day 324.95616765.

solution This represents the 324th day of the year plus 0.95616765 mean solar day. The decimal fraction in hours is $24 \times 0.95616765 = 22.948022$; the decimal fraction of this, 0.948022, in minutes is $60 \times 0.948022 = 56.881344$; the decimal fraction of this in seconds is $60 \times 0.881344 = 52.88064$. The epoch is at 22 h, 56 min, 52.88 s on the 324th day of the year.

Universal time coordinated is equivalent to *Greenwich mean time* (GMT), as well as *Zulu* (Z) *time.* There are a number of other "universal time" systems, all interrelated (see Wertz, 1984) and all with the mean solar day as the fundamental unit. For present purposes, the distinction between these systems is not critical, and the term *universal time,* abbreviation UT, will be used from now on.

For computations, UT will be required in two forms: as a fraction of a day and in degrees. Given UT in the normal form of hours, minutes, and seconds, it is converted to fractional days as

$$UT_{day} = \frac{1}{24} \left(\text{hours} + \frac{\text{minutes}}{60} + \frac{\text{seconds}}{3600}\right) \qquad (2.18)$$

In turn, this may be converted to degrees as

$$UT^{\circ} = 360 \times UT_{day} \qquad (2.19)$$

2.9.3 Julian dates[1]

Calendar times are expressed in UT, and although the time interval between any two events may be measured as the difference in their calendar times, the calendar time notation is not suited to computations where the timing of many events has to be computed. What is required is a reference time to which all events can be related in decimal days. Such a reference time is provided by the Julian zero time reference, which is 12 noon (12:00 UT) on January 1 in the year 4713 B.C.! Of course, this date would not have existed as such at the time; it is a hypothetical starting point which can be established by counting backward according to a certain formula. For details of this intriguing time reference, see Wertz (1984). The important point is that ordinary calendar times are easily converted to Julian dates, measured on a continuous time scale of Julian days. To do this, first determine the day of the year, keeping in mind that day zero, denoted as Jan 0, is December 31. For example, noon on December 31 is denoted as Jan 0.5, and noon on January 1 is denoted as Jan 1.5. It may seem strange that the last day of December should be denoted as "day zero in January," but it will be seen that this makes the day count correspond to the actual calendar day.

A general method for calculating the Julian day for any date and time is given in Duffett-Smith (1986, p. 9). The Mathcad routine based on this is illustrated in the following example.

Example 2.10 Find the Julian day for 13 h UT on 18 December 2000.

[1]It should be noted that the Julian date is not associated with the Julian calendar introduced by Julius Caesar

solution Enter the 4-digit year:

$$y := 2000$$

Enter the month number:

$$mon := 12$$

Enter the day number of the month:

$$dy := 18 \cdot day$$

Enter the time of day:

$$hours := 13 \cdot hr \qquad minutes := 0 \cdot min \qquad seconds := 0 \cdot sec$$

$$d := dy + hours + minutes + seconds$$

$$y := if\,(mon \le 2, y - 1, y)$$

$$mon := if\,(mon \le 2, mon + 12, mon)$$

$$A := floor\left(\frac{y}{100}\right)$$

$$B := 2 - A + floor\left(\frac{A}{4}\right)$$

$$C: = floor\,(365 \cdot 25\ y)$$

$$D: = floor\,(30.6001 \cdot (mon + 1)\,)$$

$$JD := B \cdot day + C \cdot day + D \cdot day + d + 1720994.5 \cdot day$$
$$JD := 2451897.0417 \cdot day$$
= = = = = = = = = = = =

In Sec. 2.9.7, certain calculations require a time interval measured in *Julian centuries,* where a Julian century consists of 36,525 mean solar days. The time interval is reckoned from a reference time of January 0.5, 1900, which corresponds to 2,415,020 Julian days. Denoting the reference time as JD_{ref}, the Julian century by *JC,* and the time in question by *JD,* then the interval in Julian centuries from the reference time to the time in question is given by

$$T = \frac{JD - JD_{ref}}{JC} \tag{2.20}$$

This is illustrated in the following example.

Example 2.11 Find the time in Julian centuries from the reference time January 0.5, 1900 to 13 h UT on 18 December 2000.

solution

$$JD_{ref}: = 2415020 \cdot day$$

$$JC: = 36525 \cdot day$$

From Example 2.10:

$$JD : = 2451897.0417 \cdot day$$

Equation (2.20) gives

$$T : = \frac{JD - JD_{ref}}{JC}$$

$$T = 1.00963838$$

= = = = = = =

Note that the time units are days and T is dimensionless.

2.9.4 Sidereal time

Sidereal time is time measured relative to the fixed stars (Fig. 2.7). It will be seen that one complete rotation of the earth relative to the fixed

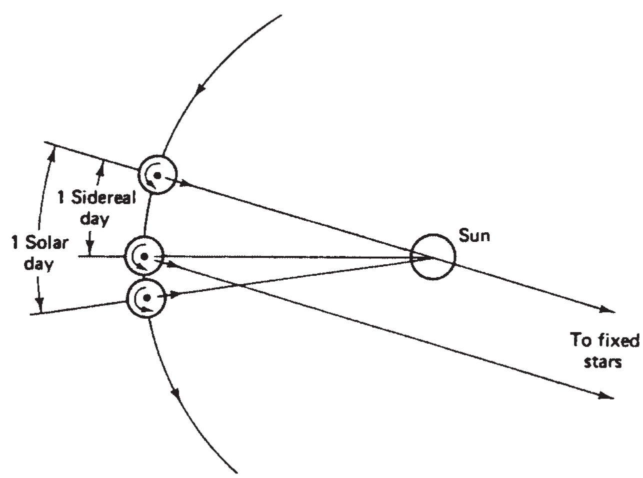


Figure 2.7 A sidereal day, or one rotation of the earth relative to fixed stars, is shorter than a solar day.

stars is not a complete rotation relative to the sun. This is because the earth moves in its orbit around the sun.

The *sidereal day* is defined as one complete rotation of the earth relative to the fixed stars. One sidereal day has 24 sidereal hours, one sidereal hour has 60 sidereal minutes, and one sidereal minute has 60 sidereal seconds. Care must be taken to distinguish between sidereal times and mean solar times which use the same basic subdivisions. The relationships between the two systems, given in Bate et al. (1971), are

$$\begin{aligned} \text{1 mean solar day} &= \text{1.0027379093 mean sidereal days} \\ &= 24^{\text{h}}\ 3^{\text{m}}\ 56^{\text{s}}.55536 \text{ sidereal time} \\ &= \text{86,636.55536 mean sidereal seconds} \end{aligned} \tag{2.21}$$

$$\begin{aligned} \text{1 mean sidereal day} &= \text{0.9972695664 mean solar days} \\ &= 23^{\text{h}}\ 56^{\text{m}}\ 04^{\text{s}}.09054 \text{ mean solar time} \\ &= \text{86,164.09054 mean solar seconds} \end{aligned} \tag{2.22}$$

Measurements of longitude on the earth's surface require the use of sidereal time (discussed further in Sec. 2.9.7). The use of 23 h, 56 min as an approximation for the mean sidereal day will be used later in determining the height of the geostationary orbit.

2.9.5 The orbital plane

In the orbital plane, the position vector **r** and the velocity vector **v** specify the motion of the satellite, as shown in Fig. 2.8. For present purposes, only the magnitude of the position vector is required. From the geometry of the ellipse (see App. B), this is found to be

$$r = \frac{a\,(1 - e^2)}{1 + e \cos \nu} \tag{2.23}$$

The true anomaly ν is a function of time, and determining it is one of the more difficult steps in the calculations.

The usual approach to determining ν proceeds in two stages. First, the mean anomaly M at time t is found. This is a simple calculation:

$$M = n\,(t - T) \tag{2.24}$$

Here, n is the mean motion, as previously defined in Eq. (2.8), and T is the time of perigee passage.

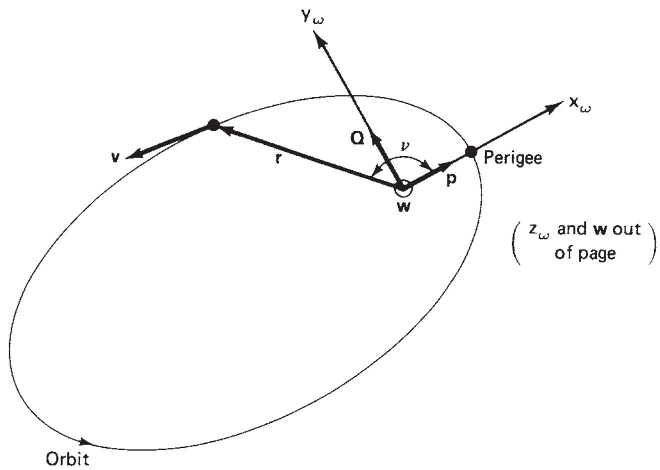


Figure 2.8 Perifocal coordinate system (**PQW** frame).

The time of perigee passage T can be eliminated from Eq. (2.24) if one is working from the elements specified by NASA. For the NASA elements,

$$M_0 = n\,(t_0 - T)$$

Therefore,

$$T = t_0 - \frac{M_0}{n} \tag{2.25}$$

Hence, substituting this in Eq. (2.24) gives

$$M = M_0 + n\,(t - t_0) \tag{2.26}$$

Consistent units must be used throughout. For example, with n in degrees/day, time $(t - t_0)$ must be in days and M_0 in degrees, and M will then be in degrees.

Example 2.12 Calculate the time of perigee passage for the NASA elements given in Table 2.1.

solution The specified values at epoch are mean motion $n = 14.23304826$ rev/day, mean anomaly $M_0 = 246.6853°$, and $t_0 = 223.79688452$ days. In this instance it is only necessary to convert the mean motion to degrees/day, which is $360n$. Applying Eq. (2.25) gives

$$\mathrm{T} = 223.79688452 - \frac{246.6853}{14.23304826 \times 360}$$

$$= \underline{\underline{223.79604425 \text{ days}}}$$

Once the mean anomaly M is known, the next step is to solve an equation known as *Kepler's equation.* Kepler's equation is formulated in terms of an intermediate variable E, known as the *eccentric anomaly,* and is usually stated as

$$M = E - e \sin E \tag{2.27}$$

This rather innocent looking equation requires an iterative solution, preferably by computer. The following example in Mathcad shows how to solve for E as the root of the equation

$$M - (E - e \sin E) = 0 \tag{2.28}$$

Example 2.13 Given that the mean anomaly is 205 degrees and the eccentricity 0.0025, calculate the eccentric anomaly.

solution

M : = 205 · deg   e : = 0.0025

E : = π   …This is the initial guess value for E.

E : = root (M − E + e · sin (E) , E)   …This is the root equation which Mathcad solves for E.

E = 204.938 · deg
= = = = = = = =

Once E is found, ν can be found from an equation known as *Gauss' equation,* which is

$$\tan \frac{\nu}{2} = \sqrt{\frac{1 + e}{1 - e}} \tan \frac{E}{2} \tag{2.29}$$

It may be noted that r, the magnitude of the radius vector, also can be obtained as a function of E and is

$$r = a\,(1 - e \cos E) \tag{2.30}$$

For near-circular orbits where the eccentricity is small, an approximation for ν directly in terms of M is

$$\nu \cong M + 2e \sin M + \frac{5}{4} e^2 \sin 2M \tag{2.31}$$

Note that the first M term on the right-hand side must be in radians, and ν will be in radians.

Example 2.14 For satellite no. 14452 the eccentricity is given in the NASA prediction bulletin as 9.5981×10^{-3} and the mean anomaly at epoch as 204.9779°. The mean motion is 14.2171404 rev/day. Calculate the true anomaly and the magnitude of the radius vector 5 s after epoch. The semimajor axis is known to be 7194.9 km.

solution

$$n = \frac{14.2171404 \times 2\pi}{86400} \cong 0.001 \text{ rad/s}$$

$$M = 204.9779 + 0.001 \times \frac{180}{\pi} \times 5$$

$$= 205.27° \quad \text{or} \quad 3.583 \text{ rad}$$

Since the orbit is near-circular (small eccentricity), Eq. (2.26) may be used to calculate the true anomaly ν as

$$\nu \cong 3.583 + 2 \times 9.5981 \times 10^{-3} \times \sin 205.27 + {}^5/_4 \times 9.5981^2 \times 10^{-6} \sin (2 \times 205.27)$$

$$= 3.575 \text{ rad}$$

$$= \underline{\underline{204.81°}}$$

Applying Eq. (2.23) gives r as

$$r = \frac{7194.9 \times (1 - 9.5981^2 \times 10^{-6})}{1 + (9.5981 \times 10^{-3}) \times \cos 204.81} \cong \underline{\underline{7257 \text{ km}}}$$

The magnitude r of the position vector $\mathbf{r}$ may be calculated by either Eq. (2.23) or Eq. (2.30). It may be expressed in vector form in the *perifocal coordinate system.* Here, the orbital plane is the fundamental plane, and the origin is at the center of the earth (only earth-orbiting satellites are being considered). The positive x axis lies in the orbital plane and passes through the perigee. Unit vector $\mathbf{P}$ points along the positive x axis as shown in Fig. 2.8. The positive y axis is rotated 90° from the x axis in the orbital plane, in the direction of satellite motion, and the unit vector is shown as $\mathbf{Q}$. The positive z axis is normal to the orbital plane such that coordinates xyz form a right-hand set, and the unit vector is shown as $\mathbf{W}$. The subscript ω is used to distinguish the xyz coordinates in this system, as shown in Fig. 2.8. The position vector in this coordinate system, which will be referred to as the **PQW** *frame,* is given by

$$\mathbf{r} = (r \cos \nu)\,\mathbf{P} + (r \sin \nu)\,\mathbf{Q} \tag{2.32}$$

The perifocal system is very convenient for describing the motion of the satellite. If the earth were uniformly spherical, the perifocal coordinates would be fixed in space, i.e., inertial. However, the equatorial bulge causes rotations of the perifocal coordinate system, as described in Sec. 2.8.1. These rotations are taken into account when the satellite position is transferred from perifocal coordinates to *geocentric-equatorial coordinates,* described in the next section.

Example 2.15 Using the values $r = 7257$ km and $\nu = 204.81°$ obtained in the previous example, express r in vector form in the perifocal coordinate system.

solution

$$r_P = 7257 \times \cos 204.81 = -6587.6 \text{ km}$$

$$r_Q = 7257 \times \sin 204.81 = -3045.3 \text{ km}$$

Hence

$$\boldsymbol{r} = -6587.6\mathbf{P} - 3045.3\mathbf{Q} \text{ km}$$

2.9.6 The geocentric-equatorial coordinate system

The *geocentric-equatorial coordinate system* is an inertial system of axes, the reference line being fixed by the fixed stars. The reference line is the line of Aries described in Sec. 2.5. (The phenomenon known as the precession of the equinoxes is ignored here. This is a very slow rotation of this reference frame, amounting to approximately 1.396971° per Julian century, where a Julian century consists of 36,525 mean solar days.) The fundamental plane is the earth's equatorial plane. Figure 2.9 shows the part of the ellipse above the equatorial plane and the orbital angles Ω, ω, and i. It should be kept in mind that Ω and ω may be slowly varying with time, as shown by Eqs. (2.12) and (2.13).

The unit vectors in this system are labeled **I, J,** and **K,** and the coordinate system is referred to as the **IJK** *frame,* with positive **I** pointing along the line of Aries. The transformation of vector **r** from the **PQW** frame to the **IJK** frame is most easily expressed in matrix form, the components being indicated by the appropriate subscripts:

$$\begin{bmatrix} r_I \\ r_J \\ r_K \end{bmatrix} = \tilde{\mathbf{R}} \begin{bmatrix} \mathrm{r_P} \\ \mathrm{r_Q} \end{bmatrix} \tag{2.33a}$$

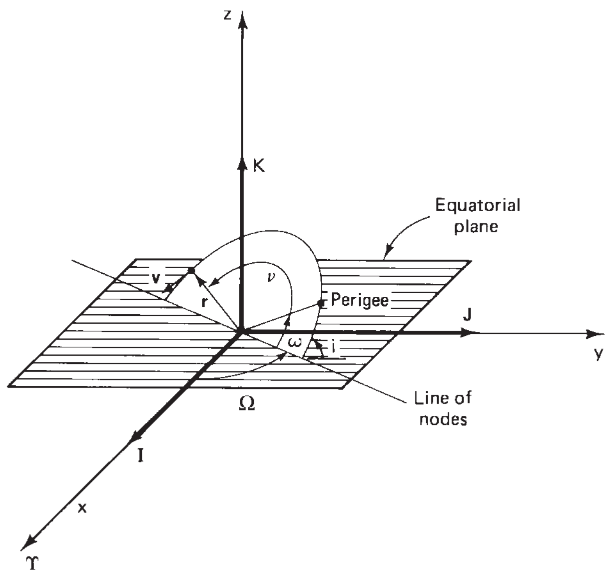


Figure 2.9 Geocentric-equatorial coordinate system (**IJK** frame).

where the transformation matrix $\tilde{\mathbf{R}}$ is given by R

$$\tilde{\mathbf{R}} = \begin{bmatrix} (\cos\Omega\cos\omega - \sin\Omega\sin\omega\cos i) & (-\cos\Omega\sin\omega - \sin\Omega\cos\omega\cos i) \\ (\sin\Omega\cos\omega + \cos\Omega\sin\omega\cos i) & (-\sin\Omega\sin\omega + \cos\Omega\cos\omega\cos i) \\ (\sin\omega\sin i) & (\cos\omega\sin i) \end{bmatrix} \tag{2.33b}$$

This gives the components of the position vector **r** for the satellite, in the **IJK,** or inertial, frame. It should be noted that the angles Ω and ω take into account the rotations resulting from the earth's equatorial bulge, as described in Sec. 2.8.1. Because matrix multiplication is most easily carried out by computer, the following example is completed in Mathcad.

Example 2.16 Calculate the magnitude of the position vector in the **PQW** frame for the orbit specified below. Calculate also the position vector in the **IJK** frame and its magnitude. Confirm that this remains unchanged from the value obtained in the **PQW** frame.

solution The given orbital elements are

$$\Omega := 300 \cdot \text{deg} \quad \omega := 60 \cdot \text{deg} \quad i := 65 \cdot \text{deg} \quad r_P := -6500 \cdot \text{km}$$
$$r_Q := 4000 \cdot \text{km}$$

$r := \sqrt{r_P{}^2 + r_Q{}^2}$ ...from Eq. (2.32)

$$r = 7632.2 \cdot km$$
$$= = = = = = =$$

Equation (2.33) is

$$\begin{bmatrix} r_I \\ r_J \\ r_K \end{bmatrix} := \begin{pmatrix} \cos(\Omega)\cdot\cos(\omega) - \sin(\Omega)\cdot\sin(\omega)\cdot\cos(i) & -\cos(\Omega)\cdot\sin(\omega) - \sin(\Omega)\cdot\cos(\omega)\cdot\cos(i) \\ \sin(\Omega)\cdot\cos(\omega + \cos(\Omega)\cdot\sin(\omega)\cdot\cos(i) & -\sin(\Omega)\cdot\sin(\omega) + \cos(\Omega)\cdot\cos(\omega)\cdot\cos(i) \\ \sin(\omega)\cdot\sin(i) & \cos(\omega)\cdot\sin(i) \end{pmatrix} \begin{pmatrix} r_P \\ r_Q \end{pmatrix}$$

$r_i = -4685.3 \cdot km$

$r_J = 5047.7 \cdot km$ ...These are the values obtained by Mathcad.

$$r_K = -3289.1 \cdot km$$

The magnitude is

$$| r | = 7632.2 \cdot km$$
$$= = = = = = = =$$

This is seen to be the same as that obtained from the **P** and **Q** components.

2.9.7 Earth station referred to the IJK frame

The earth station's position is given by the geographic coordinates of latitude λ_E and longitude ϕ_E. (Unfortunately, there does not seem to be any standardization of the symbols used for latitude and longitude. In some texts, as here, the Greek lambda is used for latitude and the Greek phi for longitude. In other texts, the reverse of this happens. One minor advantage of the former is that latitude and lambda both begin with the same *la* which makes the relationship easy to remember.)

Care also must be taken regarding the sign conventions used for latitude and longitude because different systems are sometimes used, depending on the application. In this book, north latitudes will be taken as positive numbers and south latitudes as negative numbers, zero latitude, of course, being the equator. Longitudes east of the Greenwich meridian will be taken as positive numbers, and longitudes west, as negative numbers.

The position vector of the earth station relative to the **IJK** frame is **R** as shown in Fig. 2.10. The angle between **R** and the equatorial plane, denoted by ψ_E in Fig. 2.10, is closely related, but not quite equal to, the earth station latitude. More will be said about this angle shortly. **R** is obviously a function of the rotation of the earth, and so first it is necessary to find the position of the Greenwich meridian relative to the **I** axis as a function of time. The angular distance from the **I** axis to the Greenwich meridian is measured directly as *Greenwich sidereal time* (GST), also known as the *Greenwich hour angle,* or GHA. Sidereal time is described in Sec. 2.9.4.

GST may be found using values tabulated in some almanacs (see Bate et al., 1971), or it may be calculated using formulas given in Wertz (1984). In general, sidereal time may be measured in time units of sidereal days, hours, and so on, or it may be measured in degrees, minutes, and seconds. The formula for GST in degrees is

$$\mathrm{GST} = 99.6910 + 36{,}000.7689 \times T + 0.0004 \times T^2 + \mathrm{UT}^\circ \tag{2.34}$$

Here, UT° is universal time expressed in degrees, as given by Eq. (2.19). T is the time in Julian centuries, given by Eq. (2.20).

Once GST is known, the *local sidereal time* (LST) is found by adding the east longitude of the station in degrees. East longitude for the

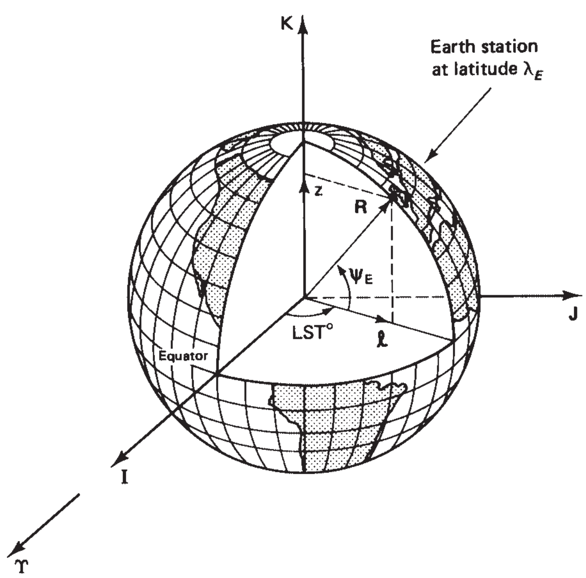


Figure 2.10 Position vector **R** of the earth relative to the **IJK** frame.

earth station will be denoted as EL. Recall that previously longitude was expressed in positive degrees east and negative degrees west. For east longitudes, EL = ϕ_E, while for west longitudes, EL = 360° + ϕ_E. For example, for an earth station at east longitude 40°, EL = 40°. For an earth station at west longitude 40°, EL = 360 + (−40) = 320°. Thus the local sidereal time in degrees is given by

$$\mathrm{LST} = \mathrm{GST} + \mathrm{EL} \tag{2.35}$$

The procedure is illustrated in the following examples

Example 2.17 Find the GST for 13 h UT on 18 December 2000.

solution From Example 2.11:

$$T := 1.009638376$$

The first three terms of Eq. (2.34) add up to

$$\mathrm{GST} := 99.6910 \cdot \mathrm{deg} + 36000.7689 \cdot T \cdot \mathrm{deg} + .0004 \cdot T^2 \cdot \mathrm{deg}$$

Note that Mathcad stores this result in radians even though the terms are in degrees:

$$\mathrm{GST} = 636.128 \cdot \mathrm{rad}$$

The universal time is

$$\mathrm{UT} := 13 \cdot \mathrm{h}$$

Converting this to a fraction of earth rotation:

$$\mathrm{UT} := \frac{2 \cdot \pi}{\mathrm{day}} \cdot \mathrm{UT}$$

This gives UT in radians:

$$\mathrm{UT} = 3.403 \cdot \mathrm{rad}$$

Hence GST in radians is

$$\mathrm{GST} := \mathrm{GST} + \mathrm{UT}$$

$$\mathrm{GST} := \mathrm{mod}\,(\mathrm{GST}, 2 \cdot \pi)$$

Using the mod function, multiple revolutions are removed, and Mathcad allows this to be expressed in degrees as

$$\underline{\underline{\mathrm{GST} = 282.449 \cdot \mathrm{deg}}}$$

Example 2.18 Find the LST for Thunder Bay, longitude 89.26°W for 13 h UT on December 18, 2000.

solution Expressing the longitude in degrees west:

$$\mathrm{WL} := -89.26 \cdot \mathrm{deg}$$

In degrees east this is

$$\mathrm{EL} := 2 \cdot \pi + \mathrm{WL} \qquad \mathrm{EL} = 270.74 \cdot \mathrm{deg}$$

GST is obtained from Example 2.17, and Eq. (2.35) gives

$$\mathrm{LST} := \mathrm{GST} + \mathrm{EL} \qquad \mathrm{LST} = 9.655 \cdot \mathrm{rad}$$

Taking mod 2π and expressing the result in degrees:

$$\mathrm{LST} := \mathrm{mod}\,(\mathrm{LST}, 2 \cdot \pi) \qquad \underline{\underline{\mathrm{LST} = 193.189 \cdot \mathrm{deg}}}$$

Knowing the LST enables the position vector **R** of the earth station to be located with reference to the **IJK** frame as shown in Fig. 2.10. However, when **R** is resolved into its rectangular components, account must be taken of the oblateness of the earth. The earth may be modeled as an *oblate spheroid,* in which the equatorial plane is circular, and any meridional plane (i.e., any plane containing the earth's polar axis) is elliptical, as illustrated in Fig. 2.11. For one particular model, known as a *reference ellipsoid,* the semimajor axis of the ellipse is equal to the equatorial radius, the semiminor axis is equal to the polar radius, and the surface of the ellipsoid represents the *mean sea level.* Denoting the semimajor axis by a_E and the semiminor axis by b_E and using the known values for the earth's radii gives

$$a_E = 6378.1414 \text{ km} \tag{2.36}$$

$$b_E = 6356.755 \text{ km} \tag{2.37}$$

From these values, the eccentricity of the earth is seen to be

$$e_E = \frac{\sqrt{a_E{}^2 - b_E{}^2}}{a_E} = 0.08182 \tag{2.38}$$

In Figs. 2.10 and 2.11, what is known as the *geocentric latitude* is shown as ψ_E. This differs from what is normally referred to as *latitude.* An imaginary plumb line dropped from the earth station makes an angle λ_E with the equatorial plane, as shown in Fig. 2.11. This is known as the *geodetic latitude,* and for all practical purposes here, this can be taken as the geographic latitude of the earth station.

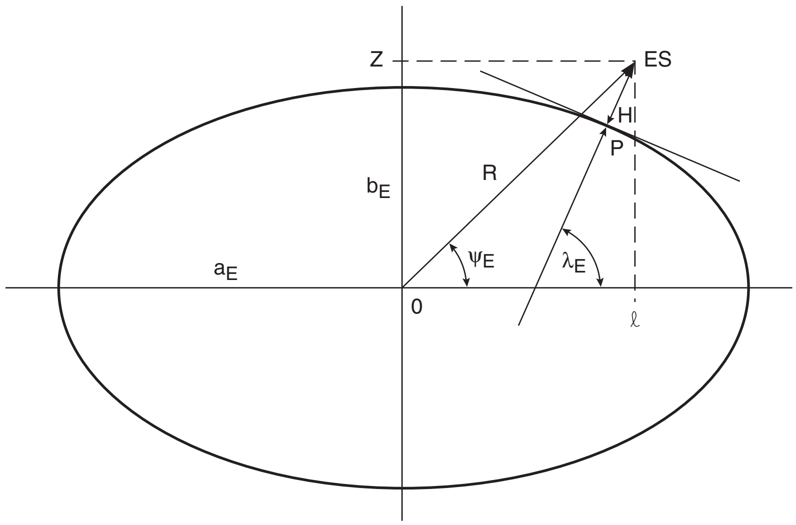


Figure 2.11 Reference ellipsoid for the earth showing the geocentric latitude ψ_E and the geodetic latitude λ_E.

With the height of the earth station above mean sea level denoted by *H,* the geocentric coordinates of the earth station position are given in terms of the geodetic coordinates by (Thompson, 1966)

$$N = \frac{a_E}{\sqrt{1 - e_E^2 \sin^2 \lambda_E}} \tag{2.39}$$

$$R_I = (N + H) \cos \lambda_E \cos \text{LST} = l \cos \text{LST} \tag{2.40}$$

$$R_J = (N + H) \cos \lambda_E \sin \text{LST} = l \sin \text{LST} \tag{2.41}$$

$$R_K = [N\,(1 - e_E^2) + H] \sin \lambda_E = z \tag{2.42}$$

Example 2.19 Find the components of the radius vector to the earth station at Thunder Bay, given that the latitude is 48.42 degrees, the height above sea level is 200 m, and the LST is 167.475·deg.

solution The given data are

$$\text{LST} := 167.475 \cdot \text{deg} \qquad \lambda_E := 48.42 \cdot \text{deg} \qquad H := 200 \cdot \text{m}$$

The required earth constants are

$$a_E := 6378.1414 \cdot \text{km} \qquad e_E := .08182$$

$$l := \left(\frac{a_E}{\sqrt{1 - e_E^2 \cdot \sin(\lambda_E)^2}} + H \right) \cdot \cos(\lambda_E) \;\ldots Eq.\ (2.40)$$

$$z := \left[\frac{a_E \cdot (1 - e_E^2}{\sqrt{1 - e_E^2 \cdot \sin(\lambda_E)^2)}} + H \right] \cdot \sin(\lambda_E) \ldots Eq.\ (2.41)$$

For check purposes, the values are

$$l = 4241 \cdot \text{km} \qquad z = 4748.2 \cdot \text{km}$$

$$R := \begin{pmatrix} l \cdot \cos(\text{LST}) \\ l \cdot \sin(\text{LST}) \\ z \end{pmatrix} \qquad \ldots\text{This gives the R components in matrix form.}$$

The values are

$$R = \begin{pmatrix} -4140.1 \\ 919.7 \\ 4748.2 \end{pmatrix} \cdot \text{km}$$

The magnitude of the R vector is

$$|\,R\,| = 6366.4 \cdot \text{km}$$

At this point, both the satellite radius vector **r** and the earth station radius vector **R** are known in the **IJK** frame for any position of satellite and earth. From the vector diagram shown in Fig. 2.12*a,* the range vector ρ is obtained as

$$\rho = \mathbf{r} - \mathbf{R} \tag{2.43}$$

This gives ρ in the **IJK** frame. It then remains to transform ρ to the observer's frame, known as the *topocentric-horizon frame,* shown in Fig. 2.12*b*.

2.9.8 The topocentric-horizon coordinate system

The position of the satellite, as measured from the earth station, is usually given in terms of the azimuth and elevation angles and the range ρ. These are measured in the *topocentric-horizon coordinate system* illustrated in Fig. 2.12*b*. In this coordinate system, the fundamental plane is the observer's horizon plane. In the notation given in Bate et al. (1971), the positive x axis is taken as south, the unit vector being denoted by **S.** The positive y axis points east, the unit vector being **E.** The positive z axis is "up," pointing to the observer's zenith, the unit vector being **Z.** (*Note:* This is not the same z as that used in Sec. 2.9.7.) The frame is referred to as the **SEZ** *frame,* which of course rotates with the earth.

As shown in the previous section, the range vector ρ is known in the **IJK** frame, and it is now necessary to transform this to the **SEZ** frame.

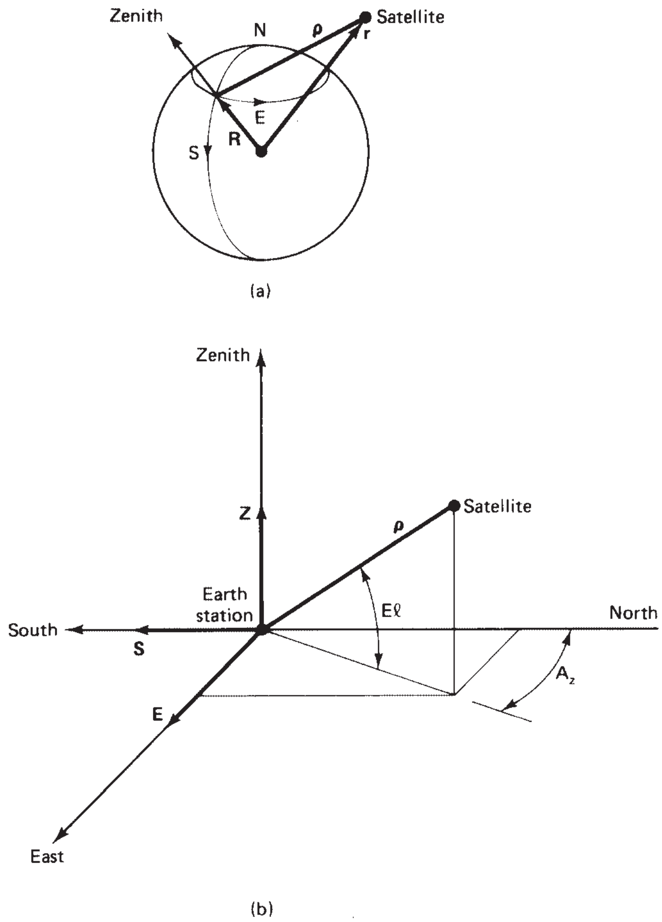


Figure 2.12 Topocentric-horizon coordinate system (**SEZ** frame): (*a*) overall view; (*b*) detailed view.

Again, this is a standard transformation procedure. See Bate et al., 1971.

$$\begin{bmatrix} \rho_S \\ \rho_E \\ \rho_Z \end{bmatrix} = \begin{bmatrix} \sin\psi_E \cos \mathrm{LST} & \sin\psi_E \sin \mathrm{LST} & -\cos\psi_E \\ -\sin \mathrm{LST} & \cos \mathrm{LST} & 0 \\ \cos\psi_E \cos \mathrm{LST} & \cos\psi_E \sin \mathrm{LST} & \sin\psi_E \end{bmatrix} \begin{bmatrix} \rho_I \\ \rho_J \\ \rho_K \end{bmatrix} \tag{2.44}$$

From Fig. 2.11, the geocentric angle ψ_E is seen to be given by

$$\Psi_E = \arctan \frac{z}{l} \tag{2.45}$$

The coordinates l and z given in Eqs. (2.40) and (2.42) are known in terms of the earth station height and latitude, and hence the range vector is known in terms of these quantities and the LST. As a point of interest, for zero height, the angle ψ_E is known as the *geocentric latitude* and is given by

$$\tan \psi_{E\,(H=0)} = (1 - e_E^2) \tan \lambda_E \tag{2.46}$$

Here, e_E is the earth's eccentricity, equal to 0.08182. The difference between the geodetic and geocentric latitudes reaches a maximum at a geocentric latitude of 45°, when the geodetic latitude is 45.192°.

Finally, the magnitude of the range and the antenna look angles are obtained from

$$\rho = \sqrt{\rho_S^2 + \rho_E^2 + \rho_Z^2} \tag{2.47}$$

$$El = \arcsin\left(\frac{\rho_Z}{\rho}\right) \tag{2.48}$$

We define an angle α as

$$\alpha = \arctan \frac{|\rho_E|}{|\rho_S|} \tag{2.49}$$

Then the azimuth depends on which quadrant α is in and is given by Table 2.2.

TABLE 2.2 Azimuth angles

ρ_S	ρ_E	Azimuth degrees
−	+	α
+	+	$180 - \alpha$
+	−	$180 + \alpha$
−	−	$360 - \alpha$

Example 2.20 The **IJK** range vector components for a certain satellite, at GST = 240 degrees, are as given below. Calculate the corresponding range and the look angles for an earth station the coordinates for which are, latitude 48.42 degrees N, longitude 89.26 degrees W, height above mean sea level 200 m.

solution Given data:

$\rho_I := -1280 \cdot \text{km}$ $\quad \rho_J := -1278 \cdot \text{km}$ $\quad \rho_K := 66 \cdot \text{km}$

$\text{GST} := 240 \cdot \text{deg}$ $\quad \lambda_E := 48.42 \cdot \text{deg}$ $\quad \phi_E := -89.26 \cdot \text{deg}$

$\text{H} := 200 \cdot \text{m}$

The required earth constants are

$$a_E := 6378.1414 \cdot km \qquad e_E := .08182$$

$$l := \left(\frac{a_E}{\sqrt{1 - e_E{}^2 \cdot \sin(\lambda_E)^2}} + H\right) \cdot \cos(\lambda_E) \text{ ...Eq. (2.40)}$$

$$z := \left[\frac{a_E \cdot (1 - e_E{}^2)}{\sqrt{1 - e_E{}^2 \cdot \sin(\lambda_E)^2}} + H\right] \cdot \sin(\lambda_E) \text{ ...Eq. (2.41)}$$

The values for check purposes are

$$l = 4241 \cdot km \qquad z = 4748.2 \cdot km$$

$$\psi_E := \operatorname{atan}\left(\frac{z}{l}\right) \qquad \text{Eq. (2.45)} \qquad \psi_E = 48.2 \cdot deg$$

$$LST := 240 \cdot deg + \phi_E \qquad \text{...Eq. (2.35)}$$

$$D := \begin{bmatrix} (\sin(\psi_E) \cdot \cos(LST)) & (\sin(\psi_E) \cdot \sin(LST)) & -\cos(\psi_E) \\ (-\sin(LST)) & (\cos(LST)) & 0 \\ (\cos(\psi_E) \cdot \cos(LST)) & (\cos(\psi_E) \cdot \sin(LST)) & (\sin(\psi_E)) \end{bmatrix}$$

$$D := \begin{pmatrix} -0.651 & 0.365 & -0.666 \\ -0.489 & -0.872 & 0 \\ -0.581 & 0.326 & 0.746 \end{pmatrix} \qquad \text{...The D-values are given for check purposes.}$$

$$\begin{bmatrix} \rho_S \\ \rho_E \\ \rho_Z \end{bmatrix} := D \cdot \begin{bmatrix} \rho_I \\ \rho_J \\ \rho_K \end{bmatrix} \qquad \text{...Eq. (2.44)}$$

$$\begin{bmatrix} \rho_S \\ \rho_E \\ \rho_Z \end{bmatrix} := \begin{pmatrix} 323 \\ 1740.6 \\ 377 \end{pmatrix} \cdot km \qquad \text{...The values are given for check purposes}$$

In Mathcad the magnitude is given simply by

$$|\rho| = 1810 \cdot km$$

= = = = = = =

$$El := \operatorname{asin}\left(\frac{\rho_Z}{\rho}\right) \qquad \text{...Eq. (2.48)} \qquad El = 12 \cdot deg$$

= = = = = =

$\alpha := \operatorname{atan}\left(\left|\frac{\rho_E}{\rho_S}\right|\right)$ ...Eq. (2.49)

The azimuth is determined by setting the quadrant conditions (see Table 2.4) as

$$Az_a := \text{if}\,[\,(\rho_S < 0 \cdot m) \cdot (\rho_E > 0 \cdot m)\,,\ \alpha,\ 0]$$

$$Az_b := \text{if}\,[\,(\rho_S > 0 \cdot m) \cdot (\rho_E > 0 \cdot m)\,,\ 180 \cdot deg - \alpha,\ 0]$$

$$Az_c := \text{if}\,[\,(\rho_S > 0 \cdot m) \cdot (\rho_E < 0 \cdot m)\,,\ 180 \cdot deg + \alpha,\ 0]$$

$$Az_d := \text{if}\,[\,(\rho_S < 0 \cdot m) \cdot (\rho_E < 0 \cdot m)\,,\ 360 \cdot deg - \alpha,\ 0]$$

Since all but one of these are zero, the azimuth is given by

$$Az := Az_a + Az_b + Az_c + Az_d \qquad Az = 100.5 \cdot deg$$

= = = = = = =

Note that the range could also have been obtained from

$$\sqrt{\rho_I^{\,2} + \rho_J^{\,2} + \rho_K^{\,2}} = 1810 \cdot km$$

2.9.9 The subsatellite point

The point on the earth vertically under the satellite is referred to as the *subsatellite point*. The latitude and longitude of the subsatellite point and the height of the satellite above the subsatellite point can be determined from a knowledge of the radius vector **r.** Figure 2.13 shows the meridian plane which cuts the subsatellite point. The height of the terrain above the reference ellipsoid at the subsatellite point is denoted by H_{SS}, and the height of the satellite above this, by h_{SS}. Thus the total height of the satellite above the reference ellipsoid is

$$h = H_{SS} + h_{SS} \tag{2.50}$$

Now the components of the radius vector **r** in the **IJK** frame are given by Eq. (2.33). Figure 2.13 is seen to be similar to Fig. 2.11, with the difference that r replaces R, the height to the point of interest is h rather than H, and the subsatellite latitude λ_{SS} is used. Thus Eqs. (2.39) through (2.42) may be written for this situation as

$$N = \frac{a_E}{\sqrt{1 - e_E^{\,2}\sin^2\lambda_{SS}}} \tag{2.51}$$

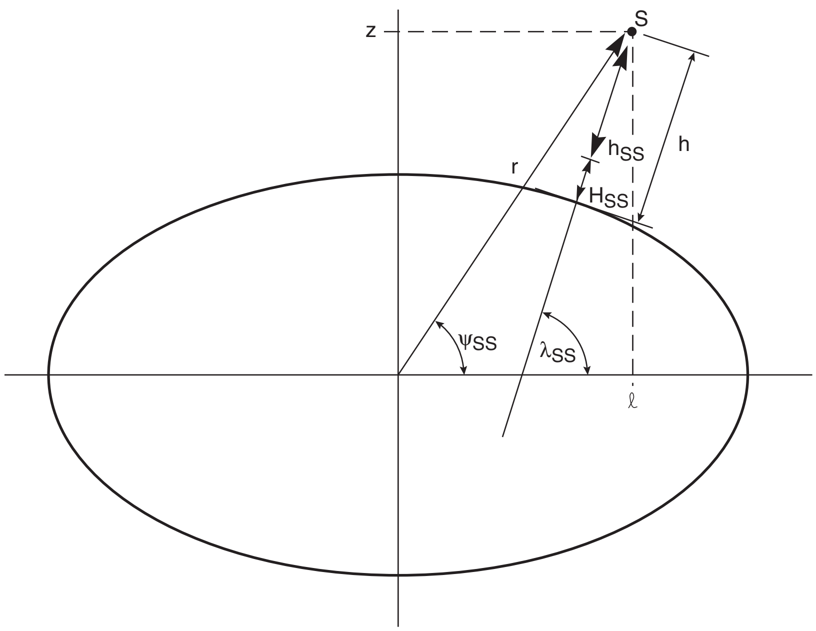


Figure 2.13 Geometry for determining the subsatellite point.

$$r_I = (N + h) \cos \lambda_{SS} \cos LST \tag{2.52}$$

$$r_J = (N + h) \cos \lambda_{SS} \sin LST \tag{2.53}$$

$$r_K = [N (1 - {e_E}^2) + h] \sin \lambda_{SS} \tag{2.54}$$

We now have three equations in three unknowns, LST, λ_E, and h, and these can be solved as shown in the following Mathcad example. In addition, by analogy with the situation shown in Fig. 2.10, the east longitude is obtained from Eq. (2.35) as

$$\text{EL} = \text{LST} - \text{GST} \tag{2.55}$$

where GST is the Greenwich sidereal time.

Example 2.21 Determine the subsatellite height, latitude, and LST for the satellite in Example 2.16.

solution From Example 2.16, the components of the radius vector are

$$\begin{bmatrix} r_I \\ r_J \\ r_K \end{bmatrix} := \begin{pmatrix} -4685.3 \cdot \text{km} \\ 5047.7 \cdot \text{km} \\ -3289.1 \cdot \text{km} \end{pmatrix}$$

The required earth constants are

$$a_E = 6378.1414 \cdot km \qquad e_E := .08182$$

In order to solve Eqs. (2.51) through (2.54) by means of the Mathcad solve-block, guess values must be provided for the unknowns. Also, rather than using Eq. (2.51), it is easier to write N directly into Eqs. (2.52) through (2.54).
Guess value for LST:

$$LST := \pi$$

Guess value for latitude:

$$\lambda_E := \operatorname{atan}\left(\frac{r_K}{r_I}\right)$$

Guess value for height:

$$h := |r| - a_E$$

The Mathcad solve block can now be used: Given

$$r_I = \left(\frac{a_E}{\sqrt{1 - {e_E}^2 \cdot \sin(\lambda_E)^2}} + h\right) \cdot \cos(\lambda_E) \cdot \cos(LST)$$

$$r_J = \left(\frac{a_E}{\sqrt{1 - {e_E}^2 \cdot \sin(\lambda_E)^2}} + h\right) \cdot \cos(\lambda_E) \cdot \sin(LST)$$

$$r_K = \left[\frac{a_E \cdot (1 - {e_E}^2)}{\sqrt{1 - {e_E}^2 \cdot \sin(\lambda_E)^2}} + h\right] \cdot \sin(\lambda_E)$$

$$\begin{bmatrix} \lambda_E \\ h \\ LST \end{bmatrix} := \text{Find}(\lambda_E, h, LST)$$

$\lambda_E = -25.65 \cdot deg$ $h = 1258 \cdot km$ $LST = 132.9 \cdot deg$
= = = = = = = =   = = = = = =   = = = = = = = =

2.9.10 Predicting satellite position

The basic factors affecting satellite position are outlined in the previous sections. The NASA two-line elements are generated by prediction models contained in Spacetrack report no. 3 (ADC USAF, 1980), which also contains Fortran IV programs for the models. Readers desiring highly accurate prediction methods are referred to this report. Spacetrack report No. 4 (ADC USAF, 1983) gives details of the models used for atmospheric density analysis.

2.10 Sun-Synchronous Orbit

Some details of the Tiros-N/NOAA satellites used for search and rescue (Sarsat) operations are given in Sec. 1.5. These satellites operate in sun-synchronous orbits. The orientation of a sun-synchronous orbit remains fixed relative to the sun, as illustrated in Fig. 2.14, the angle Φ remaining constant.

Figure 2.15 shows an alternative view, from above the earth's north pole. The angle Φ is equal to $\Omega - \alpha$ and to the local solar time expressed in degrees, as will be explained shortly. From this view, the earth rotates daily around a fixed axis in space, and the sun appears to move in space, relative to the fixed stars, because of the earth's yearly orbit around the sun. The mean yearly orbit of 360° takes 365.24 mean solar days, and hence the daily shift is $360/365.24 = 0.9856°$. The angle α, shown in Fig. 2.15 and known as the *right ascension of the mean sun,* moves eastward by this amount each day.

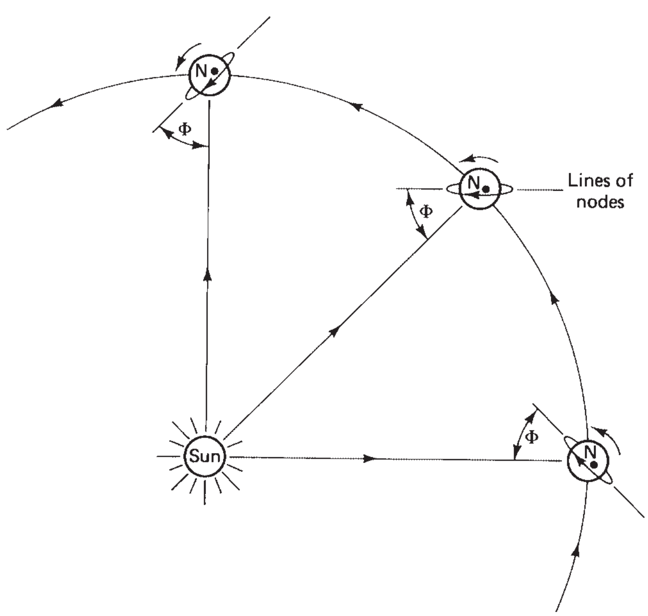


Figure 2.14 Sun-synchronous orbit.

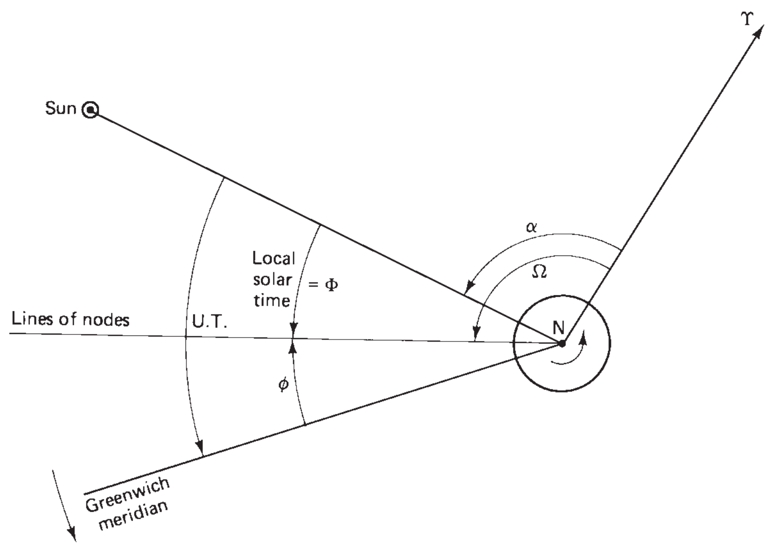


Figure 2.15 The condition for sun synchronicity is that the local solar time should be constant. Local solar time = $\Omega - \alpha$, which is also equal to the angle Φ shown in Fig. 2.14. This is the local solar time at the ascending node, but a similar situation applies at other latitude crossings.

For the satellite orbit to be sun-synchronous, the right ascension of the ascending node Ω also must increase eastward by this amount. Use is made of the regression of the nodes to achieve sun synchronicity. As shown in Sec. 2.8.1 by Eqs. (2.12) and (2.14), the rate of regression of the nodes and the direction are determined by the orbital elements *a, e,* and *i.* These can be selected to give the required regression of 0.9856° east per day.

The orbital parameters for the Tiros-N satellites are listed in Table 2.3. These satellites follow near-circular, near-polar orbits.

From Fig. 2.15 it will be seen that with sun synchronicity the angle $\Omega - \alpha$ remains constant. This is the angle Φ shown in Fig. 2.14. Solar time is measured by the angle between the sun line and

TABLE 2.3 Tiros-N Series Orbital Parameters

	833-km orbit	870-km orbit
Inclination	98.739°	98.899°
Nodal period	101.58 min	102.37 min
Nodal regression	25.40°/orbit W	25.59°/orbit W
Nodal precession	0.986°/day E	0.986°/day E
Orbits per day	14.18	14.07

Source: Schwalb, 1982a.

the meridian line, as shown in Fig. 2.15, known as the *hour angle*. For example, universal time discussed in Sec. 2.9.2 is the hour angle between the sun and the Greenwich meridian, as shown in Fig. 2.15. Likewise, local solar time is the hour angle between the sun and the local meridian. The local solar time for the line of nodes is seen to be $\Omega - \alpha$, and as shown, for a sun-synchronous orbit this is constant. In this case the latitude is zero (equator), but a similar argument can be applied for the local solar time at any latitude. What this means in practical terms is that a satellite in sun-synchronous orbit crosses a given latitude at the same local solar time and hence under approximately the same solar lighting conditions each day. This is a desirable feature for weather and surveillance satellites.

Local solar time is not the same as standard time. Letting ϕ represent the longitude of the ascending node in Fig. 2.15 gives

$$\text{Local solar time} = \text{UT} + \phi \tag{2.56}$$

As before, ϕ is negative if west and positive if east. UT is related to standard time by a fixed amount. For example, universal time is equal to eastern standard time plus 5 hours. If the correction between standard time and UT is S hours, then

$$\text{UT} = \text{standard time} + \text{S} \tag{2.57}$$

This can be substituted in Eq. (2.56) to find the relationship between local standard time and local solar time.

If a satellite in sun-synchronous orbit completes an integral number of orbits per day, it also will be earth-synchronous. This means that equatorial crossings separated in time by a 1-day period will occur at the same longitude and hence at the same standard time. As it is, the Tiros-N satellites do not have an integral number of orbits per day, and although the local solar time of crossings remains unchanged, the standard time will vary, as will the longitude. The same arguments as used here for equatorial crossings can be applied to any latitude.

The nodal regression given in Table 2.3 is the number of degrees rotated by the earth during one orbit of the satellite, and is approximately equal to 360° divided by the number of orbits per day. It clearly is different from the rate of nodal precession.

2.1 Problems

2.1. State Kepler's three laws of planetary motion. Illustrate in each case their relevance to artificial satellites orbiting the earth.

2.2. Using the results of App. B, show that for any point P, the sum of the focal distances to S and S' is equal to $2a$.

2.3. Show that for the ellipse the differential element of area $dA = r^2\, d\upsilon/2$, where $d\upsilon$ is the differential of the true anomaly. Using Kepler's second law, show that the ratio of the speeds at apoapsis and periapsis (or apogee and perigee for an earth-orbiting satellite) is equal to $(1 - e)/(1 + e)$.

2.4. A satellite orbit has an eccentricity of 0.2 and a semimajor axis of 10,000 km. Find the values of (a) the latus rectum; (b) the minor axis; (c) the distance between foci.

2.5. For the satellite in Prob. 2.4, find the length of the position vector when the true anomaly is 130°.

2.6. The orbit for an earth-orbiting satellite orbit has an eccentricity of 0.15 and a semimajor axis of 9000 km. Determine (a) its periodic time; (b) the apogee height; (c) the perigee height. Assume a mean value of 6371 km for the earth's radius.

2.7. For the satellite in Prob. 2.6, at a given observation time during a south to north transit, the height above ground is measured as 2000 km. Find the corresponding true anomaly.

2.8. The semimajor axis for the orbit of an earth-orbiting satellite is found to be 9500 km. Determine the mean anomaly 10 min after passage of perigee.

2.9. The following conversion factors are exact: one foot = 0.3048 meters; one statute mile = 1609.344 meters; one nautical mile = 1852 meters. A satellite travels in an unperturbed circular orbit of semimajor axis $a = 27{,}000$ km. Determine its tangential speed in (a) km/s, (b) mi/h, and (c) knots.

2.10. Explain what is meant by *apogee height* and *perigee height*. The Cosmos 1675 satellite has an apogee height of 39,342 km and a perigee height of 613 km. Determine the semimajor axis and the eccentricity of its orbit. Assume a mean earth radius of 6371 km.

2.11. The Aussat 1 satellite in geostationary orbit has an apogee height of 35,795 km and a perigee height of 35,779 km. Assuming a value of 6378 km for the earth's equatorial radius, determine the semimajor axis and the eccentricity of the satellite's orbit.

2.12. Explain what is meant by the ascending and descending nodes. In what units would these be measured, and in general, would you expect them to change with time?

2.13. Explain what is meant by (a) line of apsides and (b) line of nodes. Is it possible for these two lines to be coincident?

2.14. With the aid of a neat sketch, explain what is meant by each of the angles: *inclination*; *argument of perigee*; *right ascension of the ascending node.* Which of these angles would you expect, in general, to change with time?

2.15. The inclination of an orbit is 67°. What is the greatest latitude, north and south, reached by the subsatellite point? Is this orbit retrograde or prograde?

2.16. Describe briefly the main effects of the earth's equatorial bulge on a satellite orbit. Given that a satellite is in a circular equatorial orbit for which the semimajor axis is equal to 42,165 km, calculate (*a*) the mean motion, (*b*) the rate of regression of the nodes, and (*c*) the rate of rotation of argument of perigee.

2.17. A satellite in polar orbit hs a perigee height of 600 km and an apogee height of 1200 km. Calculate (*a*) the mean motion, (*b*) the rate of regression of the nodes, and (c) the rate of rotation of the line of apsides. The mean radius of the earth may be assumed equal to 6371 km.

2.18. What is the fundamental unit of universal coordinated time? Express the following universal times in (*a*) days and (*b*) degrees: 0 h, 5 min, 24 s; 6 h, 35 min, 20 s; your present time.

2.19. Determine the Julian days for the following dates and times: 0300 h, January 3, 1986; midnight March 10, 1999; noon, February 23, 2000; 1630 h, March 1, 2003.

2.20. Find, for the times and dates given in Prob. 2.19, (*a*) T in Julian centuries and (*b*) the corresponding Greenwich sidereal time.

2.21. Find the month, day, and UT for the following Julian dates: (*a*) day 3.00, year 1991; (*b*) day 186.125, year 2000; (*c*) day 300.12157650, year 2001; (*d*) day 3.29441845, year 2004; (*e*) day 31.1015, year 2010.

2.22. Find the Greenwich sidereal time (GST) corresponding to the Julian dates given in Prob. 2.21.

2.23. The Molnya 3-(25) satellite has the following parameters specified: perigee height 462 km; apogee height 40,850 km; period 736 min; inclination 62.8°. Using an average value of 6371 km for the earth's radius, calculate (*a*) the semimajor axis and (*b*) the eccentricity. (*c*) Calculate the nominal mean motion n_0. (*d*) Calculate the mean motion. (*e*) Using the calculated value for *a,* calculate the anomalistic period and compare with the specified value calculate. (*f*) the rate of regression of the nodes, and (*g*) the rate of rotation of the line of apsides.

2.24. Repeat the calculations in Prob. 2.23 for an inclination of 63.435°.

2.25. Determine the orbital condition necessary for the argument of perigee to remain stationary in the orbital plane. The orbit for a satellite under this condition has an eccentricity of 0.001 and a semimajor axis of 27,000 km. At a given epoch the perigee is exactly on the line of Aries. Determine the satellite position relative to this line after a period of 30 days from epoch.

2.26. For a given orbit, K as defined by Eq. (2.11) is equal to 0.112 rev/day. Determine the value of inclination required to make the orbit sun-synchronous.

2.27. A satellite has an inclination of 90° and an eccentricity of 0.1. At epoch, which corresponds to time of perigee passage, the perigee height is 2643.24 km directly over the north pole. Determine (*a*) the satellite mean motion. For 1 day after epoch determine (*b*) the true anomaly, (*c*) the magnitude of the radius vector to the satellite, and (*d*) the latitude of the subsatellite point.

2.28. The following elements apply to a satellite in inclined orbit: $\Omega_0 = 0°$; $\omega_0 = 90°$; $M_0 = 309°$; $i = 63°$; $e = 0.01$; $a = 7130$ km. An earth station is situated at 45° N, 80° W, and at zero height above sea level. Assuming a perfectly spherical earth of uniform mass and radius 6371 km, and given that epoch corresponds to a GST of 116°, determine at epoch the orbital radius vector in the (*a*) **PQW** frame; (*b*) **IJK** frame; (*c*) the position vector of the earth station in the **IJK** frame; (*d*) the range vector in the **IJK** frame; (*e*) the range vector in the **SEZ** frame; and (*f*) the earth station look angles.

2.29. A satellite moves in an inclined elliptical orbit, the inclination being 63.45°. State with explanation the maximum northern and southern latitudes reached by the subsatellite point. The nominal mean motion of the satellite is 14 rev/day, and at epoch the subsatellite point is on the ascending node at 100° W. Calculate the longitude of the subsatellite point 1 day after epoch. The eccentricity is 0.01.

2.30. A "no name" satellite has the following parameters specified: perigee height 197 km; apogee height 340 km; period 88.2 min; inclination 64.6°. Repeat the calculations in Prob. 2.23 for this satellite.

2.31. Given that $\Omega_0 = 250°$, $\omega_0 = 85°$, and $M_0 = 30°$ for the satellite in Prob. 2.30, calculate, for 65 min after epoch ($t_0 = 0$) the new values of Ω, ω, and M. Find also the true anomaly and radius.

2.32. From the NASA bulletin given in App. C, determine the date and the semimajor axis.

2.33. Determine, for the satellite listed in the NASA bulletin of App. C, the rate of regression of the nodes, the rate of change of the argument of perigee, and the nominal mean motion n_0.

2.34. From the NASA bulletin in App. C, verify that the orbital elements specified are for a nominal S-N equator crossing.

2.35. A satellite in exactly polar orbit has a slight eccentricity (just sufficient to establish the idea of a perigee). The anomalistic period is 110 min. Assuming that the mean motion is $n = n_0$ calculate the semimajor axis. Given that at epoch the perigee is exactly over the north pole, determine the position of the perigee relative to the north pole after one anomalistic period and the time taken for the satellite to make one complete revolution relative to the north pole.

2.36. A satellite is in an exactly polar orbit with apogee height 7000 km and perigee height 600 km. Assuming a spherical earth of uniform mass and radius 6371 km, calculate (*a*) the semimajor axis, (*b*) the eccentricity, and (*c*) the orbital period. (*d*) At a certain time the satellite is observed ascending directly overhead from an earth station on latitude 49° N. Give that the argument of perigee is 295° calculate the true anomaly at the time of observation.

2.37. For the satellite elements shown in Fig. 2.6, determine approximate values for the latitude and longitude of the subsatellite point at epoch.